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Sept 19 1886

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A SHORT

COMMENT

ON

SIR I. NEWTON's

PRINCIPIA.

CONTAINING

NOTES upon ſome DIFFICULT PLACES

OF THAT

EXCELLENT BOOK.

Magnum iter aſcendo, —— PROPERT.

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1760

THE PREFACE.

THE PRINCIPIA *being a book which is universally read by all the world, that pretend to any degree of philosophical learning; it cannot be improper to explain such passages therein as seem obscure and difficult. For altho' it is written in as clear a stile as can be done in so few words; yet, by reason of its conciseness, and the difficulty of the subjects treated on, many things occur which require some farther explication, especially to young beginners.*

Accordingly several mathematical writers have endeavoured to explain some parts or other, of this book, to make them intelligible to common readers, who, without such helps, would find it very difficult to get forward.

The noble subjects this book treats of, being no less than the grand fabric of the world, and the whole system of nature, although comprised in so little a compass, makes it highly deserving of every illustration that can be given it.

The author has clearly shewn in this book, that all the bodies in the world are actuated by the universal principle of Gravity; which is this, that every body is attracted or impelled towards any other body, by a force which is reciprocally as the square of the distance of the two bodies. On this universal principle he shews, that the motions of all the great bodies in the world are founded.

Several men had written systems of Philosophy before Sir Isaac; *but from their ignorance of nature, none of them could stand the test. But his Principles being*

built upon the unerring foundation of Observations and Experiments, must necessarily stand good, till the dissolution of nature itself.

This little Treatise was written many years since; for when I studied the Principia, I was frequently at a stop, which obliged me to make calculations here and there, as I went on; and when I had done, I set them down as notes upon these places. Wherein I only meddled with these places, that appeared difficult to me. These notes collected together are the subject of the following Comment. And I have revised the whole, and added several things that seemed wanting. Yet I believe there are some things still behind, which are not sufficiently explained by any Commentator; and especially such as are there laid down without their demonstrations.

As no body is reckoned a Philosopher that does not read the Principia; therefore I thought proper to publish this small Tract, supposing that it may be useful to others, that have a desire to read that celebrated work. What I have further to mention, is this. The passages referred to, and which are to be explained, are not taken from the Latin edition, which would not suit the English reader; but from Motte's *translation, and from the first edition thereof, if there happen to be more.*

W. Emerson.

ADVERTISEMENT.

Besides the common Algebraic characters which are in use, I make use of this $\propto$, which signifies a proportion. Thus A $\propto$ BC, signifies that A is in a constant ratio to BC, or that A is as BC.

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SHORT COMMENT,

CONTAINING

Explanations on Sir ISAAC NEWTON's Principia, as Tranflated by AND. MOTTE.

[COR. 2. to the laws. And therefore if the weight p is to the weight A, &c.] For p will have the fame effect as P, if $p : P :: pH \times OL : pN \times$ perpendicular from O on pN. And $P : A ::$ (by what went before) $KO : OL$. Therefore, *ex æquo*, if p and A are in equilibrio, it will be $p : A :: pH \times KO : pN \times$ perpendicular from O on pN. Note, the line pN ought to be drawn in fig. 2, and not PL.

[Cor. 4. ib.] This is demonftrated in Keil's Introduction, Theor. 20; as likewife in Lem. 23, p. 127, of this book.

[ib. p. 28, *fub finem*; but the diftance between thefe two centers,] that is, between the center of the two, and the center of all the reft. Further, the actions of all the bodies may be confidered as the fum of the actions of every two; and then the cafe will be plain.

[Sch. to cor. 6, p. 33, then will ST reprefent] For if RV = retardation of defcribing 2RA + 2VA, ST will be the retardation of defcribing $\frac{1}{4}$ of that, or $\frac{1}{2}$ RV + $\frac{1}{2}$VA, that is SA or FA; and therefore the body falling from S in the air, or from

Fig. T in vacuo; will have, nearly, the ſame velocity in A; the ſame of *s*A or *t*A; for aſcending to *s* in the air, or aſcending to *t* (or deſcending from *t* to A) in vacuo has the ſame velocity in A.

[ib. p. 38, but if they are turned aſide by the interpoſition] This is plain by Prop. 16, Mechanics.

[ib. And in like manner, &c.] By the ſame Prop. as before.

BOOK

BOOK I.

SECT. I.

[LEM. 10.] For let AE be divided into an infinite number of equal parts, any part, as D′ × by the velocity acquired in the time AD (that is, D × DB), is as the ſpace deſcribed in that little part of time D′ (for the ſpaces are as the times × by the velocities); and the ſum of all theſe products or areas, that is, the areas ADB, AEC, are as the whole ſpaces deſcribed in the times AD, AE; but theſe areas are as AD^2 and AE^2, by Lem. 9.

[ib. cor. 1. — to the bodies, and meaſured —] that is, and the ſaid errors meaſured by the diſtances of the bodies, &c.

[Lem. 11. have a finite curvature] Theſe words exclude thoſe curves whoſe radius of curvature is infinitely ſmall, or infinitely great.

[ib. Schol.] All this may be univerſally demonſtrated after this manner. Let AB, AF be two paraboloids. Let, latus rectum, of AF $= a$, of AB $= b$. AC $= z$. AE $= x$. CB, EF, or AD $= y$. And let m be any affirmative index, and ſuppoſe $a^m x = y^{m+1}$. And $b^{m+n} z = y^{m+n+1}$. Then $a^{\frac{m}{m+1}} x^{\frac{1}{m+1}} = y = b^{\frac{m+n}{m+n+1}} \times z^{\frac{1}{m+n+1}}$. And by involution, $a^{m \times \overline{m+n+1}} x^{m+n+1} = b^{\overline{m+n} \times \overline{m+1}} z^{m+1}$. And therefore, $a^{m^2+mn+m} x^n : b^{m^2+mn+m+n} :: z^{m+1} : x^{m+1}$. But, becauſe x is infinitely ſmall, therefore when n is affirmative, the firſt term is infinitely leſs than the ſecond; and therefore z^{m+1} is infinitely leſſer than x^{m+1}; but if 1.

Fig. 1. n is negative, the first term is infinitely greater than the second, and therefore z^{m+1} is infinitely greater than x^{m+1}, or z than x; that is, in case the first DB is infinitely less, and in case the second, infinitely greater than DF. But DB, DF are as the curvatures of AB, AF, therefore, &c.

SECT. II.

2. [Pr. 1. cor. 4. as the versed sines, of arcs] described in equal times, tending to the center of force, and bisect the chords; let AB = BC, and Bd be ⊥ to AC, then Ad = dC. And when the arch AC is diminished to infinity, e coincides with d; and consequently Ae = eC, or BeS bisects the chord AC. Complete the parallelogram ABCf, and eB = $\frac{1}{2}f$B. Note, he calls that the versed sign of an arch, which is commonly called the versed sine of half that arch.

[Pr. 4. cor. 2.] For the forces $\propto \frac{\text{velo.}^2}{\text{rad.}} \propto \frac{\text{vel}^2 \times \text{rad}}{\text{rad}^2} \propto \frac{\text{rad}}{p.\text{times}^2}$; all the other corollaries (except the last) depend on this.

[ib. cor. 8.] Let R = radius of curvature D = distance from the center of force Then will the p.time $\propto R^n \propto D^n$ by similar position), and the areas $\propto$ velocities $\propto R^{n-1} \propto D^{n-1}$. And therefore the force $\propto \frac{1}{D^{2n-1}}$: and the contrary.

3. [ibid. cor. 9.] Take the arch Bd infinitely small, and let BF be described by the revolving body, in the same time that a body falls from B to E by an uniform centripetal force, which it has at B; then cd is the space fallen in the time of describing Bd. But Cd or Bn : BE : : (as the squares of the times, that

that is, as) $Bd^2 : BF^2 :: \frac{Bd^2}{BA} : \frac{BF^2}{BA}$. But $Bn = \frac{Bd^2}{BA}$. Therefore $BE = \frac{BF^2}{BA}$. And $BE : BF : BA \ddot{\div}$. Fig. 3.

[ib. Schol. as the square of the length applied to the radius;] for, the number of reflections is $\propto$ velocity or length directly, and the radius reciprocally.

[Pr. 7. cor. 2. in the same periodical time] Let *ac* be $\parallel$ to RP, and *da* $\parallel$ to PS. Then if the periodic times be equal, the areas generated in a given infinitely small time must be equal, that is, the velocities round R and S must be reciprocally as (the $\perp$'s on PG from R and S, that is, as) RP and SG. And supposing *a*, *c*, *d*, to coincide in P, the force round R to the force round S is in the complicate ratio (of *ac* to *ad*, or) SG to SP, and the squares of the times of describing a given arch, that is reciprocally as the squares of the velocities, that is, as SG^2 to RP^2. Therefore the force round R to force round S :: is as SG^3 to $SP \times RP^2$, when the periodical times are equal. 4.

Or thus, let *p* be the place of the body when the tangent *pg* is $\parallel$ to the line RS. Then the velocities round R and S, in the place *p*, will be equal; for the small areas are equal, and their heights are equal, by reason of the parallels RS, *pg*. Draw TV, *tv*, and then by similar triangles $Sp^2 \times pv^3 = Sg^3 \times pt^3$. And $Sp^3 \times PV^3 = SG^3 \times PT^3$. Then by this prop. force round S, in P : force round S, in p :: $Sp^2 \times pv^3$ or $\frac{Sg^3 \times pt^3}{Sp}$: $SP^2 \times PV^3$ or $\frac{SG^3 \times PT^3}{SP}$.

And force round S, in *p* : force round R, in *p* :: $Sp : Rp$ or Sg.

Also

Fig. Also force round R, in p : force round R, in p : :
4. $RP^2 \times PT^3 : Rp^2 \times pt^3$ or $Sg^2 \times pt^3$: : *ex equo*, force round S, in P : force round R, in P : : $RP^2 \times PT^3$: $\frac{SG^3 \times PT^3}{SP}$: : $SP \times RP^4 : SG^3$.

[Cor. 3, in the same periodic time] for then the infinitely small and equal areas will be described in equal times in P, and both these areas and the forces will be the same, as in a circle of the same curvature with the orbit at P, and therefore the forces are the same as in the foregoing Corol.

5. [Pr. 8. Sch.] let ApD be an ellipsis, AP a circle. Then $rn : Rn$: : $(pm : Pm$: :$)$ $qn : Qn$. And by division, $qr : QR$: : $(qn : Qn$: :$)$ $CD : CA$. The force in the ellipsis $\propto \frac{qt^2 \times Sp^2}{qr}$ reciprocally $=$ (because $qr = \frac{QR \times CD}{CA}$) $\frac{QT^2 \times CA \times Sp^2}{QR \times CD}$ $=$ $\left(\text{because } \frac{QT^2}{QR} = \frac{2Pm^3}{CA^2}\right)$ $\frac{2Pm^3 \times Sp^2}{CA \times CD}$ $=$ (because $Pm^3 = \frac{pm^3 \times CA^3}{CD^3}$) $\frac{2pm^3 \times CA^2 \times Sp^2}{CD^4}$. Therefore (because CA, Sp and CD are given) the force in the ellipsis is $\propto pm^3$ reciprocally. But in the hyperbola and parabola (where CA is negative or infinite) these lines are still given, and therefore the force in any conic section is reciprocally as mp^3.

[Pr. 9. will be changed—] that is $\frac{QT^2}{QR}$ is every where the same ratio, viz. $\propto$ SP.

[ib. second way.] for PV is (by reason of the given angle at P) as the radius of curvature, that is (by reason of the similarity of the parts of the figure PQ) as SP.

[Pr. 10. second way. Add the rectangle nPv.] for $Qv^2 + nPv = Qv^2 + \overline{TP + Tv} \times \overline{TP - Tv}$
$= Qv$

$= Qv^2 + TP^2 - Tv^2 = QT^2 + TP^2 =$ square of the chord QP; and $Pv \times nV + nPv = VPv$. Fig. 5.

And if the line QV be drawn, and a circle thro' the points PQV; the triangles PQv, and PQV will be similar (the $\angle$QVP being = QPR = PQv), and therefore $Pv : PQ : PV \div$.

[Pr. 10. Sch. in the ratio of the distances from the center] for the fluxion of the ordinates is augmented or diminished in the same ratio, and that is as the force.

SECT. III.

[Pr. 13, cor. 2.] For, in the demonstrations of Prop. 11, 12, and 13, QT^2 is always equal to $QR \times$ latus rectum.

[Pr. 16, cor.] The four first corollaries are general, and agree to all conic sections; the sixth corollary belongs to one and the same parabola.

[ib. cor. 6, it is more variable] that is, in the ellipsis, the ratio of the velocity at a less and greater distance, is greater than the ratio of the square roots of the greater and lesser distance; in the hyperbola 'tis greater; for the ratio of the greater and lesser perpendiculars (which is the same with this ratio of the velocities) is greater than the ratio of the square roots of the greater and lesser distances in the ellipsis, and lesser than it in the hyperbola. For in the ellipsis the perpendiculars in the greatest and least distances are the same with these distances; and in the hyperbola, the greatest perpendicular possible is that from the focus on the assymptote, and the least, the distance to the vertex. Also (by Conics) the perpendicular $Sy \propto \sqrt{\frac{SP}{HP}}$ (fig. 3.) See Clark. (also cor. 7.)

[Pr. 17. yet greater velocity] then PH (and PK) will be negative.

[ib.

Fig. [ib. cor. 2.] The velocity in a circle is = ve-
5. locity acquired by falling thro' $\frac{1}{2}$DS, by the given centripetal force.

SECT. IV.

[Pr. 18, 19, 20.] two given points or right lines.

[Pr. 20, case 4. But because of the similar triangles VSH, *vsh*,] For (by similar Δ's SVP, *shq*) SV : SP : : *sh* : *sq* : : (by sim. Δ's, *svh*, *spq*) *sv* : *sp*. And (by the similar Δ's SHP, *shp*) SP : SH : : *sp* : *sh*; ergo SV : SH : : *sv* : *sh*; and the angle VSH = *psq* = *vsh*.

[Pr. 21. three given points or right lines.] If three tangents be given, you have 3 points Y, from which 3 equal right lines as YH are to be drawn to H, by case 3d of the last Lemma. If 2 tangents and a point P, there will be given two points Y, from which two equal lines are to be drawn to the focus H, and a third point P, from which PH is to be drawn; by case 2d Lem. In the hyperbola, it is PH — YH = SP. If three points P be given, it is done by case 1 Lem.

SECT. V.

[Lem. 17. in given angles] each to each respectively is the same invariable angle, tho' they are not all equal.

[Lem. 18. and so (by supposition)—] for *p* and *b* are in the curve, and the rectangles of the lines from *p* and P are in the given ratio.

[ib. Sch. if *p* happens to be in a right line] For if *rpq* : *spt* : : RPQ : SPT. And P be placed in *p*B; then by similar triangles *pq* : PQ : : (*p*B :
6. PB : :) *pt* : PT. And *rp* : RP : : (C*p* : CP : :) *ps* : PS. And by multiplying you have *rpq* : RPQ : : *spt* : SPT; which agrees with the Lemma. But if P is placed out of *p*B, as at *n*; then because the ratio

ratio $\frac{RnQ}{SnT}$ is greater than $\frac{RPQ}{SPT}$ it is alſo greater Fig. 6.
than $\frac{rpq}{spt}$, which is againſt the hypotheſis.

If p is placed in DC, then rpq will be $= 0$, and therefore $RPQ = 0$; and P will alſo be placed in DC.

[ib. If the two oppoſite angles] The Δ's PCR, PBT are ſimilar, for the angles at S and T are right, and C = B, being on the ſame arch PD. Therefore PR : PC : : PT : PB. And the triangles PBQ, and SCP are ſimilar, for the angles at S,Q are right, and SCP = PBA. Therefore PC : PS : : 7.
PB : PQ; and, *ex equo*, PR × PQ = PS × PT: and, *e contra*, if PR × PQ = PS × PT; the locus of the point P is a circle.

And if theſe lines are not perpendicular, yet ſince their lengths will be reciprocally as the ſines of the angles, it will follow that PQ × PR : PS × T : : ſine of S × S of T : ſine of Q × ſine of R, and the contrary.

[ib. and one or two] If the point B be ſuppoſed 8.
to move towards A and at laſt to coincide with it, ſo that AB become a tangent at B, the Lemma will ſtill hold. And if B paſs beyond A, then the figure will be converted into this BECDEAB; the Lemma will ſtill hold as before. And if B move to an infinite diſtance, then DB, AB will be parallels; and C alſo, then DC, AC will alſo be parallel; and alſo the conic ſection paſſing thro' A and D, will paſs on infinitely towards C and B.

[Lem. 19.] This may be reſolved as Prob. 12, in the Univerſal Arithmetic.

[Lem. 20.] Here is another figure relating to this Lemma (fig. 9.) 9.

[Lem. 21. Therefore (by Lem. 20) the point D] for the angles CPR, BPT, CPB are given, and therefore the lines PT, PR (to which the ſides AQ,

Fig. AQ, AS of the parallelogram are parallel) are given by position (see fig. above).

10. Here is another figure of Lem. 21. (fig. 10). This Lemma is the same with Prob. 53, of the Universal Arithmetic.

11. [Prop. 22.] This is the same with Prob. 55, in the author's Universal Arithmetic.

[Pr. 23. case 1.] This is the same with Prob. 56, of the Universal Arithmetic, or Prob. 57.

[Pr. 23. case 2. HA^2 will become to AI^2] for let n be a point in the conic section infinitely near a, thro' both which the line ib passes; then ib is a tangent at a. Then (Cor. 4. Pr. 44. B. I. my Conics) it will be, $bgd : bhd :: pgc : xby = \frac{bhd \times pgc}{bgd}$,

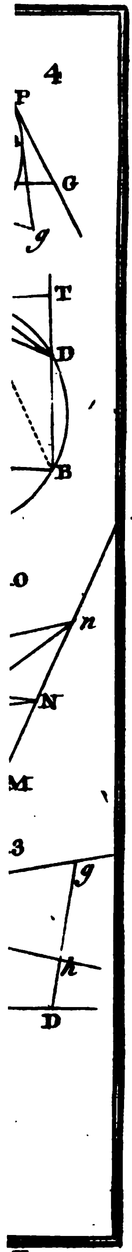
and bna or $ba^2 : ian$ or $ia^2 :: xby$ or $\frac{bhd \times pgc}{bgd} : pic$

$:: bhd \times pgc : pic \times bgd$.

12. [Pr. 24. — will (by the properties of the Conic Sections)] because any tangent may be supposed to cut the curve in two points infinitely near each other, therefore (by Cor. 4. Pr. 44. B. I. Ellipsis) these proportions follow. Also draw $kw \parallel$ to ga, to intersect pg, and let $nk^2 = tkw$. Then tkw or $nk^2 : ba^2 :: dkb : bhd :: kr^2 : br^2$. And $nk : ba :: kr : br$. And n, r, a, fall in one right line. Again, $kn^2 : ga^2 :: kp^2 : gp^2$. And $kn : gn :: kp : gp$. Whence n, p, a fall in one right line. And therefore the points p, r, a are in one right line.

13. [Lem. 22. Thus any right lines converging] Let the lines be kg, kb. Draw OB, OeD, and $pe \parallel$ to nB. Let m, q be the projected points of f and b. Then (because in the point B, OB = OD,) mn (or Bn — Bm) = fr. Also qp (or ep — eq,) : bg (or Dg — Db) :: Oe : oD :: AB : AD :: fr or mn : bg. Therefore $qp = mn$, therefore pn, qm (which are the projected lines of gr, bf,) are parallel.

[ib.



I. pa. 14.

[ib. we shall have the solution required] For the figure *bgi* supposed now to be given, may be transformed into the first figure HGI, by making as O*d* to *dg*, so is OD to DG parallel to the radius AO. Fig. 13.

[ib. For as often as two conic sections] For these conic sections being transformed into simpler ones, give the point of intersection; and thereby is had an ordinate drawn from that point of intersection in the transformed curves, corresponding to the intersection of the given curves.

[Pr. 25.] Let KG, KH be two tangents meeting in K, *yx* the third tangent meeting *ba* in *y*. Let the lines KH, KG be projected in *ki lb*; and *yb*, *yx* into *kl*, *ib*; then you have the parallelogram *bikl*; then proceed according to the proposition. 14.

[ib. For by the properties of the conic sections] by cor. 4. Pr. 44. Conics I.

[ib. But according as the points] This is plain, from the nature of the ellipsis and hyperbola; and the figure cannot be a parabola, by reason of two parallel tangents *ib* and *kl*. 15. 16.

[Lem. 23.] The Lem. is universal, as will appear by applying the demonstration to this fig. 17. 17.

[Lem. 24, from the nature of the conic sections] by Pr. 46. B. I. of my Conic Sections.

[ib. cor. 1.] This holds as well when the tangent FG is on the other side. 18.

[Lem. 25. also KH is to HL] by cor. 1. Lem. 24, for the tangents FH, LH cut the parallel tangents ML, IK in F and K.

[Lem. 25. cor. 2.] This holds as well when *qe* is on the other side of the figure; for, in all cases (by cor. 1.) $KQ \times ME$ is given wherever the points Q, E fall, as suppose in *q*, *e*: for (by cor. 1. Lem. 24.) B*q* : AM or BK : : *e*I : *e*M. And by division K*q* : BK : : MI : *e*M. And $Kq \times eM = BK \times MI =$ (by the prop.) $KQ \times ME$; and KQ : M*e* : : (K*q* : ME : :) Q*q* : E*e*. 19.

[ib.

Fig. [ib. cor. 3.] for since $eM : ME :: QK : Kq$.
19. Therefore (by Lemma 23.) if the right lines eQ, MK, and Eq be drawn, the points of bisection will be placed in a right line given in position.

[Pr. 27. Sch. describe the circle BKGC.] for the angle BKC (which is equal to the sum of the given angles PBK and KCP) is always given.

[ib. and when those other legs CK, BK] For it must be observed, that when the lines BP, CP, touch the curve at an infinite distance, that these lines are parallel to one another, and to the assymptote. Then to know the position of the assymptote; as the lines Bk, Ck revolve round the circle, the intersection k will sometime fall into the line MN, as at the point N, then BN is parallel to one assymptote. For the intersection must necessarily fall in the line MN. And for the same reason the line BM will be parallel to the other assymptote. And therefore if the angle between them be bisected by the line OH (which is done by the perp. OH), then that line is the greater axis, or parallel to it.

[ib. Sch. But the squares of the axes] For the angle NBM = angle between the assymptotes = NLM, and NLH = half the angle of the assymptotes. Therefore LH is to HN, as the transverse to the conjugate. And the squares of these axes, are as LH^2 to HN^2, or as HN^2 to HK^2, that is, as LH to HK.

20. [ib. There are also other Lemma's] For if the sections are similar, and in similar position, and concentric; the tangent acb in c, is parallel to the tangent dxe in x, and therefore ab the ordinate is bisected in c the point of contact.

[Lem. 26. cor.] This is the same with Prob. 32, of the Universal Arithmetic.

[Lem. 27. cor. in the construction] for then $IH : HF :: (IX : XY ::) ib : bf$. And $IH : HG ::$ (iL :

(iL : LM : :) ib : bg. Or, if it be made as iL : : LM : : IH : HG, it will be ib : bf : : IH : HF; therefore on the contrary, if it be ib : bf : : (iE : EV : : iX : XY : :) IH : HF. It will be iL : LM : : IH : HG, which comes to the former conſtruction. For the ſolution of Prop. 22, 23, 24, 25, 26, 27; ſee Prop. 70, 71, 72, 73, 74, 75, B. III. my Conic Sections. Fig. 20.

[Pr. 29. Sch.] Make alſo KA to AS, and LT to AT, as HG to GF, and draw MS, NA. Then the figures SAKM and ATLN are ſimilar to FGHI, and ſince three of the angles S, A, K or A, T, L are in the proper lines CB, ED, DB; if the fourth angle M or N, was in the fourth line EC, the problem would be rightly conſtructed. Therefore it is plain, its place can be no where, but where the line MN interſects EC as at i, which is the place of the angle I. 21.

Now we are to prove that PQ cuts BA in f where F is to be placed. The triangle FGI is ſimilar to PEi (by conſtruction), and ſuppoſe them ſimilar to fgi; then the triangles Pfi and Egi are alſo ſimilar; for the angles at i are equal, and the ſides, about theſe angles proportional; therefore the angle Egi = Pfi, and ſince goi = Qof, oQf or PQE will be = fig. So that to have fig ſimilar to FIG, PQF muſt interſect Eg in Q, to make the $\angle$ PQE = FIG, and the reſt follows of courſe.

SECT. VI.

[Pr. 30, Cor. 1.] For the times are as the areas, that is, as $\frac{4}{3}$GH $\times$ AS to $\frac{2}{3}$ $\times$ AS $\times$ 2AS.

[ib. Cor. 2.] For wherever the point P falls, viz. infinitely near A, ſtill $\frac{4}{3}$GH $\times$ AS (= area APS) = $\frac{1}{2}$AP $\times$ AS. And 8GH = 3AP. Therefore GH or the velocity of H : AP or the velocity of P : : 3 : 8. But the velocity of H is every where

 the

Fig. the ſame, for always $\frac{1}{2}GH \times AS =$ area ASP. And GH $\propto$ ASP $\propto$ time of deſcribing AP.

[ib. Cor. 3.] For AP is the chord of a circle paſſing thro' A, S, P; and whoſe center is H.

[Pr. 31. as GK the difference] For when F comes to touch the line GH, the point A will be diſtant towards G, from the line FQO (which will then be $\perp$ to GH) by the ſine of AOR or AOQ; and is then at L; therefore GK = arc GF — ſine of the arc AQ. And GK is as the time, and ſo it was in the conſtruction; therefore the point P is rightly found.

22. [ib. Sch. but ſince] This is demonſtrated in Keil's
23. Aſtron. lectures, pa. 289, 297; or thus. Let AON $=$N. Since (AB : SH : :) OQ : OS : : 57.29578 : B. Therefore B = OS in degrees of the circle AQ. And ſince (rad : ſine of AOQ : :) SO : SF : : B : D. Then will SF = D in degrees. Let q be the true place of the body; Q the aſſumed place. Now ſince the time is as the area SAQ = OAQ + SOQ $= \overline{QA + SF} \times \frac{OA}{2}$; therefore the time is as AQ $\pm$ SF. And therefore nearly as AQ $\pm$ D, but accurately as Aq $\pm$ SF. Take Nφ = D. Now OE : OQ : : (LE or SE — SF or) Nq $\pm$ Nφ or $q\varphi$ or Qφ — qQ : Qq. And Qφ : Qq : : QE : OQ, by compoſition, and becauſe QE = OQ $\pm$ OE. But by conſtruction $OQ = \frac{OS \times L}{R}$. And rad. (R) : coſ. of AOQ : : SO : OF or OE, therefore OE $= \frac{SO \times \text{coſ. } AOQ}{R}$. Wherefore Q$\varphi$: Qq : : (QE =) $\frac{OS \times L}{R} \pm \frac{SO \times \text{coſ. } AQ}{R} : \frac{OS \times L}{R}$: : L $\pm$ coſ. AQ : L. But N — AOQ + D = (AN — AQ + Nφ =) Qφ. Therefore Qq = E, and AOq = AOQ + QOq = AOQ + E, nearly, and therefore

fore E is rightly found. And repeating the same work with these new angles, there will be found the angles F, G, H, I. Fig. 22. 23.

[ib. the area AIKP will be given] by Prop. 87. cor. 2. and Schol. my Conic Sections, B. II. Also AIKP = OPA, by cor. 1. Prop. 86. ib.

Also area described = ASQ = ASP + PSQ = ASP + PQ × ½SN (nearly) = A (by supposition) therefore A − ASP = ½SN × PQ; and $PQ = \frac{2A - 2ASP}{SN}$, nearly.

[ib. and by such computations.—But the particular calculus] with the radius 1 and center H describe the circle RSE; and draw SP, Sp, HPT, Hpf. Let fall the perpendiculars pt, pu, upon the lines HP, SP; which will be equal, because the angles pPt (HPB), and pPu, are equal. 24.

Let the arch RT = z. Sine TQ = s, cos. HQ = x, SP = y, HP = v, AO or OB = a, SO or OH = n, OD = c. T = mean anomaly, l = latus rectum.

By similar sectors, HT (t) : HP (v) :: Tf ($\dot{z}$) : pt or $pu = v\dot{z}$. And the area $\frac{SP \times pu}{2} = \frac{vy\dot{z}}{2} =$ fluxion of BPS. But (cor. 2. Prop. 23, ellipsis) $vy = cc + \frac{nn}{cc} \times PM^2$, and (cor. 3. Prop. 72, ib.)

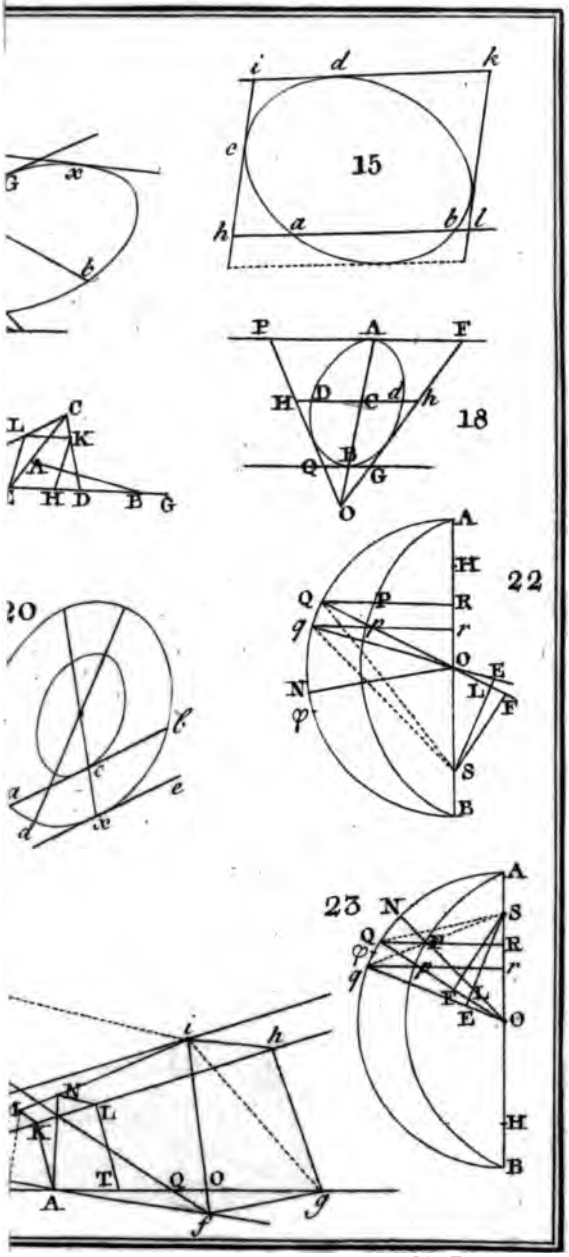
HP or $v = \frac{cc}{a + nx}$, and rad (1) : PH $\left(\frac{cc}{a + nx}\right)$ :: S : PM $= \frac{ccs}{a + nx}$, therefore $vy = cc + \frac{nnccss}{\overline{a+nx}^2}$,

and area S$Pp = \frac{1}{2}cc\dot{z} + \frac{\frac{1}{2}nnccss\dot{z}}{aa + 2anx \;\&c.} = \frac{1}{2}cc\dot{z} + \frac{nnccss\dot{z}}{2aa} - \frac{n^3c^2s^2x\dot{z}}{a^3}$. But $s\dot{z} = -\dot{x}$, and $x\dot{z} = \dot{s}$,

Fig. 24. and $\frac{cc}{a} = \frac{1}{2}l$; therefore the area $SPp = \frac{1}{4}al\dot{z} - \frac{\frac{1}{4}nnls\dot{x}}{a} - \frac{n^3ls\dot{s}s}{2aa}$, and the fluent $BSP = \frac{alz}{4} - \frac{nnl}{4a} \times$ area EHQT $- \frac{n^3ls^3}{6aa}$; and corrected, $BSP = \frac{alz}{4} + \frac{nnl}{4a} \times$ area RTQ $- \frac{n^3ls^3}{6aa}$. But $RTQ = \frac{1}{2}z - \frac{1}{2}xs$. Whence $BSP = \frac{1}{4}alz + \frac{\frac{1}{4}nnl}{a} \times \overline{\frac{1}{2}z - \frac{1}{2}xs} - \frac{n^3ls^3}{6aa} = \frac{2aa+nn}{8a}lz - \frac{nnls}{aa} \times \overline{\frac{ax}{8} + \frac{nss}{6}}$. And dividing, $z - \frac{8nns}{2a^3+ann} \times \overline{\frac{ax}{8} + \frac{nss}{6}} = T$, the mean anomaly. Whence $z = T + \frac{nn}{4aa+2nn} \times S.2T + \frac{4n^3}{6a^3+3ann} \times \overline{S.T}^3$, because T is nearly $= z$, and $2xs = S.2T$. But since n is very small by supposition, $z = T + \frac{nn}{4aa} \times S.2T + \frac{2n^3}{3a^3} \times \overline{S.T}^3$. Where the quantities $\frac{nn}{4aa} \times S.2T$, and $\frac{2n^3}{3a^3} \times \overline{S.T}^3$ are small arches to be added to T.

Now $D = c - \frac{cc}{a} = \frac{c}{a} \times \overline{a-c}$; and $D \times \overline{AO + OD} = \frac{c}{a} \times \overline{aa - cc} = \frac{cnn}{a} = nn$ nearly, because $c = a$ very near. Therefore $s.Y : \text{rad}\ (1) :: nn : 4aa$, and $s.Y$ or Y (in small arches) $= \frac{nn}{4aa}$; also $V : Y :: S\,T : \text{rad}\ (1)$, and $V = Y \times$

S.2T



Pl. II. pa. 20.

$S.2T = \frac{nn}{4aa} \times S.2T$, which is our first term, or his first equation. Fig. 24.

Again, $S.Z : \text{rad}(1) :: 4nD$ or $\frac{4nc}{a} \times \overline{a-c} :$ $3AO^2$ or $3aa$, and $S.Z = \frac{4nc}{3a^3} \times \overline{a-c}$. But $\overline{a+c} \times \overline{a-c} = aa - cc = nn$, and $a - c = \frac{nn}{a+c} = \frac{nn}{2a}$ nearly, therefore $S.Z = \frac{4nc}{3a^3} \times \frac{nn}{2a} = \frac{2n^3c}{3a^4} = \frac{2n^3}{3a^3}$, nearly. Also $X : Z :: \overline{S.T}^3 : \text{rad}(1)$. And $X = Z \times \overline{S.T}^3 = \frac{2n^3}{3a^3} \times \overline{S.T}^3$, which is the second term, or his second equation. And when T is above 90°, then S.2T, and V will be negative, and $<BHP = T + X \pm V$.

He calls, Y or $\frac{nn}{4aa}$ the greatest first equation, because it is greatest when $S.2T = 1$, and $T = 45°$. And Z or $\frac{2n^3}{3a^3}$ the greatest second equation, because the greatest it can be is when $\overline{S.T}^3 = 1$, or $T = 90°$.

SECT. VII.

[Pr. 33, to AB the principal semi-diameter] this should be $\frac{1}{2}$AB.

[ib. cor. 2.] for the two first terms of the prop. are in a ratio of equality, and so the two last.

[Pr. 34. For (by cor. 7. prop. 16.)] and cor. 6. prop. 4.

[Pr. 35. The same things supposed] to wit, that the space CS is as the area SDE of the circle, rectangled hyperbola, or parabola.

Fig. [Prop. 37, as appears by prop. 34.] and cor. 7.
25. pr. 16.

[Pr. 38, acquire the velocity CD.] by prop. 10, cor. 2, The periodical times of the ellipsis AD and AP are equal. The time of describing AP is (as APS, that is, as ADS or) as AD. Let the ellipsis AP coincide with AC, and the time of describing AC will still be as AD. Further, draw cd || to CD; because the time of describing the whole ellipses ADd, APp are equal; therefore in equal times they describe areas which are as the whole ellipses, that is, as CD to CP, or ADS to APS. Wherefore in the time D describe AD or Dd, P describes AP or Pp, and (when P coincides with C), C describes AC or Cc; therefore the velocity of D : to velocity of C : : is as Dd : Cc : : or as SD to CD; but SD and the velocity of D is given; therefore the velocity of C is as CD.

[Pr. 39. cor. 2, 3.] In these cor. the line PD is the space the body would ascend to, or fall from, (to acquire the velocity it is projected with) by a uniform centripetal force, according to cor. 1.

SECT. VIII.

[Pr. 40. cor. 1.] This is evident, by supposing ITK convex towards C.

26. [ib. cor. 2.] for by fluxions, let AC $=$ P. CD $=$ A $= x$. Then DG $\propto x^{n-1}$ (by hyp. and pr. 39.) and DEFG $=$ fluxion of the area $\propto x^{n-1}\dot{x}$. And the area DCbG $\propto \frac{x^n}{n}$. And when x becomes $= p$, the area ACbB $\propto \frac{p^n}{n}$. Therefore ADGB $\propto \frac{p^n - x^n}{n}$. And therefore by prop 39, the velocity in

in D, at the diſtance A or x, is $\propto \sqrt{\frac{P^n - x^n}{n}}$, or Fig. 26.
(becauſe n is given, and $x = A$) as $\sqrt{P^n - A^n}$.

N. B. If the force is reciprocally as the diſtance, the curve BGb will be a rectangled hyperbola, to the aſſymptotes AC, Cb; for DG $\propto \frac{1}{CD}$ = (ſuppoſe to) $\frac{bb}{CD}$. And DG $\times$ CD $= bb$, which is the known property of the hyperbola. And the velocity at C will be infinite, for the area ABbC is infinite. And the velocity at any place D is as the hyperbolic area ABGD, which may be found by Stone's Fluxions, p. 54; or by Cor. 2. Prop. 87. B. II. my Conic Sections.

[Pr. 41.—in the Trajectories found.] One being given to find the other.

[ib. given the circle VR] that is, CV is known.

[ib. in the leaſt given time,] theſe ſmall parts of time are taken conſtant and invariable.

[ib. and the triangle ICK] that is, the triangle will be invariable.

[ib. and ſuppoſe the magnitude of Q,] This quantity is any conſtant quantity, but unknown. Suppoſe in ſome caſe $\sqrt{ABFD} = b \times IK$, and Z $= b \times KN$. And in all caſes (by pr. 40.) $\sqrt{ABFD} = b \times IK$; and in all caſes Z or $\frac{Q}{A} = b \times KN$, that is in all caſes $Q = b \times A \times KN$; which is plain, becauſe Q and A $\times$ KN are conſtant quantities, therefore if it be once $\sqrt{ABFD}$: Z : : KI : KN, or $b \times IK : b \times KN$: : KI : KN; it will always be ſo.

In this prop. the line CE is indetermined; and if Q were known, the areas of the curves abz, and acx might be known to any diſtance CE.

Fig. [ib. cor. 3.] In fig. 4. Newt. Let CA $= a$ CV $= r$, CD $= x$, DF $= y$, $\sqrt{aa - rr} = n$.

Then the force DF being as $\frac{1}{x^3}$, the fluxion of the area ABFD is $\frac{-\dot{x}}{x^3}$, and the fluent as $\frac{1}{2x^2}$, and corrected, the fluent is $= \frac{aa - xx}{2aaxx} =$ area ABFD. Therefore the velocity at D is as $\frac{\sqrt{aa-xx}}{ax}$. Then,

1. Suppose a infinite, and $x = r$, then the area $\sqrt{\text{ABFD}}$ becomes $\frac{\sqrt{aa}}{ar} = \frac{1}{r}$. And since at V, the orbit IV is perp. to CV, therefore IK $=$ KN, and $\sqrt{\text{ABFD}} = \text{Z} = \frac{\text{Q}}{x}$, that is, $\frac{1}{r} = \frac{\text{Q}}{r}$, and $\text{Q} = 1$, and $\text{Z} = \frac{1}{x}$; also $\sqrt{\text{ABFD}} = \frac{\sqrt{aa-xx}}{ax}$ $= \frac{a}{ax} = \frac{1}{x}$ at any place I, and since $\sqrt{\text{ABFD}}$ $\left(\frac{1}{x}\right) : \text{Z} \left(\frac{1}{x}\right) :: \text{IK} : \text{KN}$; therefore TK$=$KN, and IN $= o$; therefore in this case the orbit is a circle, as VXR.

2. If a be less than infinite, and since at V, the orbit is perp. to CV as before, therefore IK$=$KN, and $\sqrt{\text{ABFD}} = \frac{\text{Q}}{r}$, that is, $\frac{\sqrt{aa-rr}}{ar} = \frac{\text{Q}}{r}$, or $\frac{n}{ar} = \frac{\text{Q}}{r}$, and $\text{Q} = \frac{n}{a}$. Whence $\text{Z} = \frac{n}{ax}$. Therefore Db $= \frac{n}{a} \times \frac{1}{\sqrt{\frac{aa-xx}{aaxx} - \frac{nn}{aaxx}}} = \frac{n}{2a}\sqrt{\frac{aa''x}{rr-xx}} =$

nx

$\frac{nx}{2\sqrt{rr-xx}}$. And $Dc = \frac{rr}{nx} \times Db = \frac{nrr}{2x\sqrt{rr-xx}}$, Fig. 26.
and flux of the area $VacD = \frac{nrr\dot{x}}{2x\sqrt{rr-xx}} =$ fluxion of the ſector VCX, and the Fl. : $\frac{nrr\dot{x}}{2x\sqrt{rr-xx}}$ = Sector VCX, which is as the angle VCX.

But, putting ſemiconjugate $= c$, $CH = Z$, the hyperbolic ſector VCR = Fl. : $\frac{rc\dot{z}}{2\sqrt{zz-rr}}$, and CT $= \frac{rr}{z} = x$, and $z = \frac{rr}{x}$. Then inſtead of z and $\dot{z}$, putting their values; we ſhall have the hyp. ſector VCR = Fl. : $\frac{-rrc\dot{x}}{2x\sqrt{rr-xx}} = \frac{c}{R} \times$ cir. ſector VCX. Therefore when $x =$ CT, the hyp. ſector VCR : cir. ſector CVX : : $c : n$; that is, becauſe the angle VCX is as the ſector CVX, the angle VCX is to the hyper. ſector VCR in a given ratio. And ſince CT or x continually decreaſes, as the ſector VCR increaſes, the body P draws continually nearer the center C.

3. If the velocity be greater than falling from an infinite height; the flux area ABFD = $\frac{+\dot{x}}{x^3}$, and the fluent $\frac{-1}{2xx}$. But at firſt the area (ſuppoſe) $= \frac{bb}{2}$, and $x = a$ an infinite line; therefore ABFD $- \frac{bb}{2} = \frac{1}{2aa} - \frac{1}{2xx}$, and ABFD $= \frac{bb}{2} + \frac{xx-aa}{2aaxx}$. Therefore $\sqrt{\text{ABFD}}$ is as $\sqrt{\frac{bbaaxx+xx-aa}{aaxx}}$ = (by ſubſtitution) $\sqrt{\frac{sxx-aa}{aaxx}}$.

But

Fig. 26. But in V, $x = r$, and $\sqrt{ABFD} = \sqrt{\frac{srr - aa}{aarr}} = \frac{Q}{r}$, and $Q = \frac{\sqrt{srr - aa}}{a}$, and $Z = \frac{Q}{x} = \frac{\sqrt{srr - aa}}{ax}$.

Then $Db = \frac{\sqrt{srr - aa}}{2a\sqrt{\frac{sxx - aa}{aaxx} - \frac{srr - aa}{aaxx}}} = \sqrt{\frac{srr - aa}{s}}$ $\times \frac{x}{2\sqrt{xx - rr}} =$ (by substitution) $\frac{Ax}{2\sqrt{xx - rr}}$, and $Dc = \frac{rr}{xx} \times Db = \frac{Arr}{2x\sqrt{xx - rr}}$; therefore $\frac{Arr\dot{x}}{2x\sqrt{xx - rr}} =$ flux. area $VacD$, or of VCX. And the Fl. : $\frac{Arr\dot{x}}{2x\sqrt{xx - rr}} =$ sector VCX, which is as the angle VCX.

27. But in the ellipsis VRS, let $CH = z$, semiconjugate $= c$. Then the elliptic sector VCR $=$ Fl. : $\frac{rc\dot{z}}{2\sqrt{rr - zz}}$, and $CT = \frac{rr}{z} = x$, and $z = \frac{rr}{x}$, and $\dot{z} = \frac{-rr\dot{x}}{xx}$. Then putting for z and $\dot{z}$ their values, and the elip. sector VCR $=$ Fl. : $\frac{-crr\dot{x}}{2x\sqrt{xx - rr}}$ $= \frac{c}{A} \times$ cir. sector VCX; therefore when $x = CT$, the ellip. sector VCR : cir. sector VCX : : c : A; that is, because the angle VCX is as the sector VCX; the angle VCX is to the elliptic sector VCR in a given ratio. And since CT or x continually increases, as the sector VCR increases, the body P goes continually further and further from the center C.

[ib. and the Centripetal Force becoming centrifugal] for then the curve VPQ will turn upwards;

wards; and the law of the force being the ſame, it will be conſtructed the ſame way by the elliptic ſectors; taking the point A between C and V. And here the velocity will increaſe, as it recedes from the center C. But it can never revolve round this center. Fig. 27.

SECT. IX.

[Pr. 44. or in antecedentia with a celerity] for then the line *mn*C falls beyond *s*, *sr* being $= rk$, and $sCk = 2rCk$; and conſequently the point *m* falls without the circle.

[ib. and with a leſs force] if the orbit moves ſlower in antecedentia, than with twice the celerity of CP in conſequentia; for then the point *m* falls within the circle, between *r* and *s*.

The meaning of this Prop. is this, that the difference of the forces at different diſtances from the center, are reciprocally as the cubes of the diſtances (which forces are requiſite to make the body move in a quieſcent or revolving orbit; and the diſtances to be the ſame in both.)

[ib. cor. 1.] For *mn* repreſents the difference of the forces by which *p* revolves to *n*, or P to K in equal times; and the verſed ſine of RK repreſents the force whereby a body moves from R to K in the circle, in the ſame time.

[ib. cor. 2. as half the latus rectum] for theſe forces are as *bn* or *ar* to *bp* or *as*. But $ar = \frac{nr^2}{rD} = \frac{ba^2}{aD}$. And as $D : sp^2 :: aD : 2R$; and as $D = \frac{sp^2 \times aD}{2R} = \frac{ab^2 \times aD}{2R}$; and $as = \frac{ab^2 \times aD}{sD \times 2R} = \frac{ab^2}{2R}$. Whence theſe forces are as $\frac{ba^2}{aD}$ to $\frac{ba^2}{2R}$, or as R to *a*C or CV in fig. 2. pl. 18.

[ib.

Fig. 27. [ib. cor. 3.] For let the force in the immoveable ellipsis be $\frac{FFA}{T^3}$, and in V it will be $\frac{FFT}{T^3}$ or $\frac{FF}{TT}$. And the force by which a body may revolve in a circle at the distance CV is $\frac{FFR}{T^3}$. And the difference of the forces in V (of revolving in the moveable and immoveable ellipsis) is $\frac{GGR-FFR}{T^3}$, and in every altitude A is $\frac{GGR-FFR}{A^3}$; and the force in the moveable ellipsis will be $\frac{FFA}{T^3}$ + $\frac{RGG-RFF}{A^3}$.

[ib. cor. 4.] After the same manner as in the two foregoing Corollaries; because when the velocity is given, the force $\propto$ curvature or reciprocally $\propto$ radius of curvature; therefore T : R :: $\frac{VFF}{TT}$: $\frac{RVFF}{T^3}$, which is the force wherewith a body may revolve in a circle at the distance CV. And by cor. 1. FF : GG — FF :: $\frac{RVFF}{T^3}$: $\frac{RVGG-RVFF}{T^3}$, which is the difference of the forces (in the moveable and immoveable orbits) in the vertex, V. And by the prop. $\frac{1}{T^3}$: $\frac{1}{A^3}$ :: $\frac{RVGG-RVFF}{T^3}$: $\frac{RVGG-RVFF}{A^3}$, which is the difference of forces in every altitude A; and therefore

fore the force in the moveable trajectory *upk* will 27.
be $X + \frac{RVGG-RVFF}{A^3}$.

[ib. cor. 6.] In this cor. $X = o$, and RVGG — RVFF is compounded of all given quantities, and the force in V*pk* will then be $\propto \frac{1}{A^3}$. This also appears from the Prop. for the difference of the forces in P and *p* is as $\frac{1}{Cp^3}$, but the force in one of them, viz. at P in the line VP is *o*; and therefore the other force in V*pk* is the difference of the forces, and is as $\frac{1}{Cp^3}$. Also this curve is the same with that in cor. 3. Pr. 41. for let *vad* be a circle; and 28.
by this cor. 6. $\angle vcp \propto \angle vcz \propto$ arch *va* $\propto$ area *vca* $\propto$ area *vcb*, which is the construction in cor. 3. Pr. 41; also $cp = cz$ or *cx*, because *vz*, *ax*, and *bx* are tangents, which is also the same construction as in cor. 3. Pr. 41.

[Pr. 45, but orbits acquire the same figure] Let 29.
the small parts of the curve *df*, *fb*, *bp*, &c. be described by a body A in indefinitely small given parts of time. And let another body B go from D in the same direction *de*, and with a velocity which is to the velocity of the former in the subduplicate ratio of the centripetal force of B to that of A; and let it arrive at *s* in the aforesaid small given part of time. Now since *ef* : *rs* : : force of A : force of B : : velocity^2 of A : vel.^{2} of B : : de^2 : dr^2 : : square of the time of B's moving thro' *de* : square of the time of B's moving through *dr*. Therefore, by Lem. 10, the body B will pass thro' *f*. And since *rs* is every where as rd^2, it is evident the same curve *dsf* will be described by both bodies, and which *defg* touches in *d*, *f*. Also the velocity of A in *f* : vel. of A in *d* : : as perpendicular from C or *de* :

Fig. 29. de : to ⊥ on fg : : vel. of B in f : vel. of B in d. And vel. of A in f : velocity of B in f : : vel. A in d : vel. of B in D : : de : dr; that is in a given ratio; that is fg : to fk, and bn to bl, and px to pt are in a given ratio; and therefore when A is arrived at g, B would be at k, and since $gb : ki :: gf^2 : kf^2 :: ed^2 : rd^2 :: ef : rs$; or $gb : ef :: ki : rs$; that is, the centripetal of A to that of B at equal distances being in a given ratio, B will pass thro' i, and consequently thro' b. After the same manner it may be proved, that the body B will pass thro' the points m, p, u, y, and describe the same curve with the body A.

Further, if another body D move thro' Br ∥ to de, and with a velocity which is to the velocity of A as CB to Cd, and be acted upon by a force which in the points β, δ, z, &c. is to the force in the points d, f, b, &c. respectively, as Cβ to Cd, Cδ to Cf, Cz to Cb, &c. Then I say the curve described by the body D, viz. $\beta\delta z\varphi\omega$ will be similar to the curve $dfbpy$; for in the time that A would arrive at e, D would arrive at γ; and because $C\gamma : Ce :: C\beta : Cd ::$ force at β : force at $d :: \gamma\delta : ef :: Cr - \gamma\delta$ or $C\delta : Ce - ef$ or Cf; and $C\beta : C\delta :: Cd : Cf$; $\gamma\delta$ is every where as $\gamma\beta^2$ in all the points between β and γ (by Lem. 10.) it is manifest the fig. C$\beta\delta$ is similar to the fig. Cdf; and the tangent $\delta\varepsilon$ ∥ to fg; and because the areas $Cgf : Cfd :: C\varepsilon\gamma : C\delta\beta$; wherefore when A comes to g, D comes to ε; but $gb : \varepsilon z :: Cf : C\delta :: Cg : C\varepsilon :: Cb : Cz$, as before; and $Cf : Cb :: C\delta : Cz$, whence the fig. Cδz is similar to Cfb; and after the same manner it will be proved, that the nascent figures C$z\varphi$, C$\varphi\omega$, are similar to Cbp, Cpy; and therefore the whole figure $\beta\delta z\varphi\omega$ is similar to the figure $dfbpy$.

Or universally, if the orbits $\beta\delta\varphi$, and dfp are similar. $C\beta \propto \gamma\delta \propto$ force × time^2 of descri-

bing

bing $\beta\delta \propto$ force $\times \frac{\beta\delta^2}{\text{velocity}^2} \propto$ force $\times \frac{C\beta^2}{\text{velocity}^2}$. Fig. 30.
Therefore velocity$^2 \propto$ force $\times$ distance $C\beta$. Whence if $C\beta$ be given, velocity $\propto \sqrt{\text{force}}$; and if velocity $\propto C\beta$, force $\propto C\beta$ alſo, which agrees with what went before. If the force be given, velocity $\propto \sqrt{C\beta}$. If velocity be given, force $\propto \frac{1}{C\beta}$.

[ib. Exam. 1. l. 3.] here $\frac{T^3-3TTX+3TX^2-X^3}{A^3}$ is twice repeated in the Engliſh, which is wrong.

[ib. By this collation of the terms] for ſince the ſame figure will be deſcribed by making the centripetal force proportional at equal diſtances (altering the velocity in the ſubduplicate ratio of the force); and ſince, when R is nearly $=$ T; and x very ſmall, the centripetal force in the revolving ellipſis will be as RGG — FFx, and in this orbit (becauſe in both, the denominator A^3 is the ſame) is as $T^3 - 3TTx$, therefore theſe forces ought to be proportional at all diſtances, viz. when $x = o$, and any other indeterminate diſtance A or T — x; wherefore (in theſe two caſes) RGG : T^3 : : RGG — FFx : $T^3 - 3TTx$: : (and by diviſion) FFx : $3TTx$: : FF : $3TT$, which is the conſtruction of the problem. After the ſame manner in example 2d, the centripetal force in the revolving ellipſis, and this new orbit, will be as RGG—FFx, and $T^n - nT^{n-1}x$, and to have the orbits ſimilar, putting $x = o$, RGG : T^n : : (putting T — x for any indeterminate diſtance) RGG — FFx : $T^n - nT^{n-1}x$: : (by diviſion) FFx : $nT^{n-1}x$: : FF : nT^{n-1}. After the ſame manner in Examp. 3, it will be RGG : $bT^m + cT^n$: : (RGG — FFx : $bT^m + cT^n - mbT^{m-1}x - ncT^{n-1}x$) : : FF$x$: $mbT^{m-1}x + ncT^{n-1}x$. Or GG : $bT^{m-1} + cT^{n-1}$: :

Fig. : : FF : $mbT^{n-1} + ncT^{n-1}$. The quantity $\frac{FF}{AA} + \frac{RGG-RFF}{A^3}$ (in cor. 2. pr. 44.) is universally as the centripetal force, whether the apsides move backwards or forwards, for G : F : : VCp : VCP : : (p. 180.) pCn : pCk. And the force in the revolving ellipsis is greater than in the immoveable one, when n (fig. 2.) is without the line ks; and in that case (F is less than G, and) $+ \frac{RGG-RFF}{A^3}$ will be affirmative. But the force is less when n falls between k and s, for then (F is greater than G, and) $+ \frac{RGG-RFF}{A^3}$ is negative, as it ought to be.

SECT. X.

[Pr. 50. it is evident from the construction] Since VP touches the curve in P, and PB is $\perp$ thereto, therefore B is the point of contact of the circle AD, and the wheel or generating circle, and therefore BV = AO. Also since WT is $\perp$ to TV, therefore V is the point of contact of the generating circle (whose diameter is VW) and OS, whence VW = OR. The figures are similar, because their axes are as the radii of the spheres.

[Pr. 52. hence since in unequal—] suppose TR
30. to be a smaller oscillation, to find the velocity in I; take st : SR : : TI : TR; then velocity at R in the arc TR : velocity at R in SR : : TR : SR; and velocity at I in TR : velocity at R in TR : : (Prop. 51.) velocity at t in SR : velocity at R in SR : :) $\sqrt{SR^2 - tR^2}$: SR. And *ex equo* velocity at I in TR : velocity at R in SR : : TR$\sqrt{SR^2 - tR^2}$: SR^2 : : $\frac{TR}{SR}\sqrt{SR^2 - tR^2}$: SR : : (because SR : TR : : tR : IR as) $\sqrt{TR^2 - IR^2}$: SR.

[ib.

[ib. There are obtained from the times given] for in two unequal arches, two correſponding parts are deſcribed in equal times (by Prop. 51.) therefore the velocities in theſe points will be as the entire arcs; and therefore both the velocities and arches will be known. Fig.

[ib. And if the abſolute force of any globe] that is, the force at a given diſtance be called V; then the force at the diſtance CO is CO × V. But in a given time, HY is as the force; whence CO × V is as HY, and therefore HY is deſcribed in a given time.

[ib. cor. 1. For this time] for then AR = AC, and V is given; therefore, this time : time of $\frac{1}{2}$ oſcillation : : $\sqrt{\frac{AC}{AC}} : \sqrt{\frac{AR}{AC}} :: 1 : \sqrt{\frac{AR}{AC}}$.

[ib. cor. 2. But in that caſe] for (in fig. 2. pl. 19.) by cor. 1, 2, pr. 49. AS : Bv : : PS : PV : : 2CE : CB (in this caſe as) : : 2 : 1 : : (and by diviſion) AP : BV — PV; whence AP = 2BV — 2PV; but verſed ſine of $\frac{1}{2}$PB $= \frac{BV - PV}{2}$, for coſine $= \frac{1}{2}$PV; and $\frac{1}{2}$VB — $\frac{1}{2}$VP = verſed ſine.

[ib. as M. Huygens] all this is demonſtrated in Keil's Philoſophy. See my large book of Mechanics, Prop. 40. cor. 4.

[Pr. 56, let the projection] T is projected into P and t into p.

[ib. as alſo its poſition] The greater axis 2Tt is perpendicular to PO.

[ib. and ſince the area POp] for let $\frac{\text{area } OPp}{OP} = sp$, which is given ($sp$ being ⊥ to PO); and PB^2 : Pr^2 : : PB^2 — SP^2 : PS^2; whence Ps is given, and thence SO, the angle POp, Op and Pp, the point p, and angle OPp. 31.

S E C T. XI.

[Pr. 57, and about each other] conſidering either body as at reſt.

Fig. [Pr. 58, by the ſame forces there may be deſcribed, &c.] that is, by making the velocity to the former velocity as ($\sqrt{sp}$ to $\sqrt{CP}$, that is as) $\sqrt{P + S}$ to $\sqrt{S}$, as appears by the Prop.

[Pr. 60, in a ratio ſeſquiplicate] it ſhould be ſubſeſquiplicate.

[ib. of the other ellipſis,] As $S^{\frac{1}{3}} : \overline{S + P}^{\frac{1}{3}}$. Let $S^{\frac{1}{3}} : a : e : \overline{S + P}^{\frac{1}{3}}$ be $\div$. Then $S^{\frac{1}{3}} : \overline{S + P}^{\frac{1}{3}} ::$ ($S : a^3 ::$) $e^3 : \overline{S + P}$. But (ſince $\overline{S + P} : e^3 :: a^3 : s$) e^3 is the firſt of two mean proportionals between $S + P$ and S.

[Pr. 61.] Note, if a body were placed in the center of gravity, and whoſe force would be ſufficient to cauſe one of the bodies to revolve around it, and to deſcribe the ſame figure; yet it would not cauſe the other body (if it were unequal) to deſcribe its figure, except in that law of centripetal force, which is as the diſtance where the periodic times are all equal. For, to preſerve the ſame motion as before, either body muſt be attracted to the body in the center of gravity, with a force which is as the other body, or as its own diſtance, which is the caſe of one body attracting another; and holds only (in the caſe of their being attracted by a third body in the center of gravity) in the law of centripetal force before-mentioned. And therefore in

[Pr. 62.] The foregoing note is to be obſerved.

[Pr. 64. This would be the caſe] For this only cauſes the bodies T, L, to revolve more ſwiftly round their center of gravity D, but affects not the other bodies.

[ib. with equal periodical times.] By the foregoing (in the three bodies S, T, L) the body S deſcribes an ellipſis round C; and, by conſidering the center of gravity of S, T, as deſcribing an ellipſis (as before of T, L,) it will appear the ſame way that the body L deſcribes an ellipſis round C; and as the center of gravity of S, L, deſcribes an

ellipſis,

ellipsis, so T describes an ellipsis also, round C. Fig.
Further, from what went before, the centripetal force of S towards C, is as $\overline{T+L} \times \overline{SC+CD}$, or $\overline{T+L+S} \times SC$. Also the centripetal force of T towards D, is as $L \times \overline{TD+DL} + S \times TD$ (which latter part arises from the resolution of the force ST into SD, DT, and DT acts towards D), or as $\overline{T+L+S} \times TD$. Now by cor. 2, 8. pr. 4. The periodic times are in the subduplicate ratio of the radii directly, and the subduplicate ratio of the forces inversly: therefore the periodic time of S, round C : periodic time of T, round D : : $\sqrt{\dfrac{SC}{SC \times \overline{T+L+S}}} : \sqrt{\dfrac{TD}{TD \times \overline{T+L+S}}}$; and are therefore equal; so that after one revolution, the bodies all return to their first places.

After the same manner the point C, and a fourth body V, as also the bodies S, T, L, will describe ellipsis round their common center of gravity B, for any one, and the center of gravity of the other three will describe ellipses; and the case is the same if there were more bodies. Also, as before, and by a like resolution of forces, since the forces VS, VT, VL, are resolved into VC, CS; VC, CT; VC, CL; the former acting in the direction of CV towards V, the latter to C their center of gravity; and are as the distances of the bodies therefrom, as before; they will therefore still move round their center of gravity C as before, but swifter, altho' all the four revolve round B. Also periodic time of V round B : periodic time of S round C : :

$$\sqrt{\frac{BV}{\overline{S+T+L} \times CB + BV}} : \sqrt{\frac{SC}{\overline{T+L} \times S \times SC + V \times SC}}$$

: : (that is, as) $\sqrt{\dfrac{BV}{\overline{T+L+S+V} \times BV}}$:

$\sqrt{\dfrac{SC}{\overline{T+L+S+V} \times SC}}$; and are therefore equal;

Fig. and if more bodies were added, all the bodies would perform their revolutions in equal periodic times. *Q. E. D.*

[Pr. 66. cor. 2. Such is the force NM.] This force always acts from P in a direction parallel to TS, and from M towards T.

[ib. cor. 6. the periodic time will be increased,] for then the periodic time is $\propto \frac{\text{Rad}^{\frac{3}{2}}}{\sqrt{\text{force}}}$, which is a greater ratio than $\text{Rad}^{\frac{3}{2}}$, because the force is diminished.

[ib. cor. 7. the upper apsis to go backwards] for $\frac{n^2}{m^2} - 3 > -2$. And $\frac{n^2}{m^2} > 1$; therefore n is $>$ than m. Also put D = distance (r = a given quantity,) and the force is $\left(\frac{D^3 + r^3}{D^2}\right.$, or rather) as $\frac{1}{D^2 + \frac{1}{D}}$, and when the distance increases, the decrease of the force is $D^2 + \frac{1}{D}$, which is less than $D^2 + \frac{D^3}{D}$, which is the decrease according to the duplicate ratio. Also at the conjunction, &c. the force of P towards T is, the centripetal force of T + LM — TM, or the centripetal force of T - KL. Also the decrease of the force is as $d^2 - \frac{1}{d}$, which is greater than $d^2 - \frac{d^3}{d}$, or the decrease according to the duplicate ratio. And $\frac{n^2}{m} - 3 < -2$; and $n < m$.

[ib. cor. 7. The truth of this] for then the further P recedes from T the more it is attracted towards the bodies S, S, and therefore less towards T, than it would otherwise be.

[ib.

[ib. cor. 8. when the apsides are in the syzyges] Fig.
for then NM is greater, and LM less, than before.

[ib. cor. 9. Now therefore —] For the ratio of KL to LM is least when the apsides are in the quadratures, and greatest in the syzyges.

[ib. For the forces LM] as is shewn in cor. 7.

[ib. cor. 10, from the syzyges to the quadratures] for after the syzyges, the body P by the action of S, is made to move in lines which successively cut the plane TPS at greater and greater angles. And for the same reason at the 90° from the octants (between C and A), the body is drawn into lines (or little planes) which cut TPS at greater and greater angles, which before 90° cut it at less and less.

[ib. and by a like reasoning] for the inclination 32.
is increased from u to D, diminished from D to S, increased from S to C, diminished from C to u; as is plain by supposing T and any point p wherein the body is, to be joined; and the plane pTt to revolve round pT, till it pass thro' the new place of the body p, (which it acquires either above or below the plane pTt,) by the (action of S upon it, or the) force NM; and then it will appear, how that new acquired plane cuts the plane SCT, whether in a greater or lesser angle

[ib. cor. 11.—from the former plane CD] Since the inclination of the orbit is diminished from C to A, the intersection with TSE will move from C to B, and from D to A; and seeing that inclination is increased from A to D, the intersection (or nodes) will still move towards B and towards A.

[ib. being always either retrograde] that is, considering a whole revolution; for in some points they go forward.

[ib. cor. 13. And since the causes and proportions] to one another, and to the force of S, &c.

Fig. [ib. cor. 14. But ſince the forces] For SK : LM : : accelerating force of T towards S : perturbating force of S. And therefore the perturbating force of S $= \frac{\text{LM} \times \text{accelerating force of T towards S}}{\text{SK}}$. But ſince SK = ST nearly, and LM (in its mean quantity) = PT. And the accelerating force of T towards S, is as the body S directly, and ST^2 reciprocally; therefore the perturbating forces LM, NM $\propto \frac{\text{PT} \times \text{body S}}{\text{ST}^3}$, that is, $\propto \frac{\text{PT} \times \text{bodies}}{\square \text{ periodic time of T round S}}$. Or $\propto$ PT $\times$ denſity of S $\times$ cub. apparent diameter of S. (for the apparent diameter is as the real diameter directly, and the diſtance reciprocally)

[ib. cor. 16, ſquare of the periodical time of the body P conjunctly.] This holds as true in the ſynodic time of the body P, (ſince the action of the forces that cauſe theſe errors begin and end at the quadratures, which comes to the ſame, as if they begun and ended at the ſyzyges) ſee what follows.

[ib. and hence the angular errors] theſe in one revolution are as the linear errors directly, and the radius or diſtance reciprocally, that is, as the forces and ſquare of the time of revolution of P, directly, and the diſtance TP reciprocally; that is (becauſe the force $\propto$ TP) as ſquare of the time of revolution.

[ib Let theſe ratio's] The angular errors of P obſerved from T are as $\frac{\text{forces} \times \text{time}^2}{\text{TP}}$, that is (by cor. 14.) as $\frac{\text{PT} \times \text{body S} \times \text{time}^2}{\text{PT} \times \square \text{ T's periodic time}}$, that is, in the time of one revolution of P (and if S be given), as $\frac{\square \text{ time of P's revolution}}{\square \text{ periodic time of T}}$.

[ib.

[ib. both these motions will be as the periodical time of the body P, directly,] for the angular motion or velocity, or the mean angular errors, are as the sum of all the angular errors in any time directly, and the time reciprocally; that is, (in the periodic time) as Fig. 32.

$$\frac{\text{body } S \times \square \text{ time of P's revolution}}{\text{time of P's revolution} \times \square \text{ periodic time of T}},$$

that is, as body S × time of P's revolution directly, and the square of the periodic time of T, reciprocally.

[Cor. 20. And thence the greatest height of the water] This is explained by Worster in his (Experimental) Philosophy, page 72. Also see my Geography, Prop. 12. Sect. I.

SECT. XII.

[Pr. 79. As the lineola Dd,] when the radius of the sphere is given; and when the radius is not given, as Dd and the radius conjunctly; therefore AS, and also PS or PE, may be of any magnitude.

[ib. the sum of all the rectangles PD × Dd] that is, as $\frac{PF + PD}{2} \times DF$, or $\frac{PF + PD}{2} \times \overline{PF - PD}$, or $\frac{PF^2 - PD^2}{2}$.

[Pr. 80. The whole force of the sphere will be as the whole area ANB.] Also it is manifest, that the attraction or force of any part of the sphere FEB, is as the correspondent part of the area DNB.

[Pr. 81. Ex. 1.] The area SL into AB = SL × AB, because both SL and AB are given. And the area LD into AB is a trapezoid, whose base is AB, and the two parallel sides LA and LB; for as the point D removes nearer A, the ordinate LD

Fig. 32. decreases (from B where it is LB) to A, (where it is LA); and that area is $\frac{\overline{LA+LB}}{2} \times AB$, or $\frac{\overline{LA+LB}}{2} \times \overline{LB-LA}$, that is, $\frac{LB^2-LA^2}{2}$; or it is $\frac{2LA+AB}{2} \times AB$, that is $LS \times AB$. And the ordinate $\frac{ALB}{LD}$, is the ordinate of an hyperbola between the assymptotes, whose center is L, and one assymptote LB; for ALB is a given rectangle, and the nature of the hyperbola is that $\frac{ALB}{LD}$ (LD the part of the assymptote being taken as the axis) is as the ordinate erected on D; whence, the reason of the construction will be evident; for *a*AB*b* is the aforesaid trapezoid, by construction; also in the hyperbola *ab*, the rectangle $LB \times Bb$ is given, but that (by construction) is = the rectangle ALB; when the hyperbolic area *a*AB*b* is that described by the ordinate $\frac{ALB}{LD}$. And the difference of that and the trapezoid (which is the difference of the two first areas) is the area *aba* required.

[ib. Ex. 2. the third $\frac{ALB \times SI}{2LD^2}$) this is easily calculated by fluxions, LD is a flowing quantity; the fluxion of the area is $\frac{ALB \times SI}{2} \times LD^{-2} \times \dot{LD}$. And the whole area is $\frac{ALB \times SI}{-2} \times LD^{-1}$, that is $\frac{ALB \times SI}{-2LD}$, or (when LD = LB) $\frac{ALB \times SI}{-2LB}$: from this take the area (when LD = LA, *viz.*)

ALB

$\frac{ALB \times SI}{-2LA}$, and the difference is the area sought Fig.

$\frac{ALB \times SI}{2LA} - \frac{ALB \times SI}{2LB}$.

[ib. Ex. 3.] The fluxion of the first area is $\frac{SI^2 \times SL}{\sqrt{2SI}} \times LD^{-\frac{3}{2}} \times L\dot{D}$, and the whole area is $\frac{2SI^2 \times SL \times LD^{-\frac{1}{2}}}{-\sqrt{2SI}}$, or $\frac{2SI^2 \times SL}{-\sqrt{2SI} \times \sqrt{LB}}$, and that part of the area on AB is $\frac{2SI^2 \times SL}{\sqrt{2SI}} \times \overline{\frac{1}{\sqrt{LA}} - \frac{1}{\sqrt{LB}}}$. The fluxion of the second area is $\frac{SI^2}{2\sqrt{2SI}} \times LD^{-\frac{1}{2}} \times L\dot{D}$; and the whole area $\frac{SI^2}{\sqrt{2SI}} \times LD^{\frac{1}{2}}$, and the area on AB is $\frac{SI^2}{\sqrt{2SI}} \times \overline{\sqrt{LB} - \sqrt{LA}}$. Also the fluxion of the 3d area is $\frac{SI^2 \times ALB}{2\sqrt{2SI}} \times LD^{-\frac{5}{2}} \times L\dot{D}$; and the whole area $\frac{SI^2 \times ALB}{-3\sqrt{2SI}} \times LD^{-\frac{3}{2}}$; and the area on AB is $\frac{SI^2 \times ALB}{3\sqrt{2SI}} \times \overline{\frac{1}{\sqrt{LA^3}} - \frac{1}{\sqrt{LB^3}}}$.

[ib. And those after due reduction] LA, LI, LB are ∺; for $SP = \frac{SA^2}{SI}$. $PI = \frac{SA^2}{SI} - SI$. $LI = \frac{SA^2 - SI^2}{2SI}$. And $LA = PS - AS - LI = \frac{SA^2 + SI^2}{2SI} - SA$. And $LB = \frac{SA^2 + SI^2}{2SI} + SA$. But $\frac{SA^2 + SI^2}{2SI} - SA : \frac{SA^2 - SI^2}{2SI} : \frac{SA^2 + SI^2}{2SI} +$ SA,

Fig. SA, are $::$, as will appear by multiplying them. Hence $\sqrt{LB}-\sqrt{LA}=\sqrt{2SI}$. For $2\sqrt{LB\times LA}=2SI=2LS-\overline{2SI}=LB+LA-2SI$. And $LB+LA-2\sqrt{LB\times LA}=2SI$. And (by Evolution) $\sqrt{LB}-\sqrt{LA}=\sqrt{2SI}$. Therefore the first area $\left(=\frac{2SI^2\times SL}{\sqrt{2SI}}\times\frac{\sqrt{LB}-\sqrt{LA}}{\sqrt{LB\times LA}}\right)=\frac{2SI^2\times SL}{LI}$. And the second area $=SI^2$. And $\sqrt{LB^3}-\sqrt{LA^3}$ $(=\overline{LB+LA}\times\overline{\sqrt{LB}-\sqrt{LA}}+\sqrt{LB}\times\sqrt{ALB}-\sqrt{AL}\times\sqrt{ALB}=2LS\times\sqrt{2SI}+\sqrt{ALB}\times\sqrt{2SI}=\overline{2LS+LI}\times\sqrt{2SI})=\overline{3LI+2SI}\times\sqrt{2SI}$. Whence the third area $=\frac{SI^2\times ALB}{3\sqrt{2SI}}\times$

$$\frac{\sqrt{LB^3}-\sqrt{LA^3}}{\sqrt{ALB^3}}=\frac{SI^2\times\overline{3CI+2SI}}{3\sqrt{ALB}=3LI}=SI^2+\frac{2SI^3}{3LI}.$$

[Pr. 83. let us suppose] it should be, let that superficies be not a purely, &c. as it is in the original.

SECT. XIII.

[Pr. 90. cor. 1, 2.] for the fluxion of the area is as $D^{-n}\dot{D}$, and the area as $\frac{D^{1-n}}{1-n}$ or as $\frac{1}{D^{n-1}}$; and the area ALIH, as $\frac{1}{PA^{n-1}}-\frac{1}{PH^{n-1}}$.

[Pr. 91. cor. 1. And the other part $\frac{PF}{PR}$] for the fluxion of that area is $=\frac{PF}{PR}\times P\dot{F}=\frac{PF}{\sqrt{RF^2+PF^2}}\times P\dot{F}=\overline{RF^2-PF^2}^{-\frac{1}{2}}\times PF\times P\dot{F}$. And the area $=\overline{RF^2+PF^2}^{\frac{1}{2}}=PR$. And (the area on PB

PB is PE—AD; and on PA, PD—AD. And Fig.
their difference) PE—PD, is the area on AB.

[Pr. 91. cor. 2.] This is calculated in the Appendix, by the help of the Quadratures of Curves, which see in Harris's Lex. vol. 2.

And the curve KRM is a conic section; for the indetermined quantities rise only to two dimensions. Let SA $= r$. SC $= c$. BE $= u$. PB $= b$.

$$ER^2 = PD^2 = PE^2 + ED^2 = \overline{b-u}^2 + \frac{cc}{rr} \times \overline{2ru-uu} = bb + \overline{\frac{2cc}{r} - 2b} \times u + \overline{1 - \frac{cc}{rr}} \times uu.$$

[Sch. Pr. 93, and I suppose the force proportional.] 33.
For consider A, B, as flowing quantities, and let $\dot{A}$ be given $= o$. Now since $A^{\frac{m}{n}} = B$; therefore when A becomes $A + \dot{A} = A + o$, B becomes $B + \dot{B}$, and then $B + \dot{B} = \overline{A + o}^{\frac{m}{n}} = A^{\frac{m}{n}} + \frac{m}{n} oA^{\frac{m-n}{n}} + \frac{m^2 - mn}{2n^2} ooA^{\frac{m-2n}{n}} +$ &c. Now if any quantity $\overline{A + o}^{n}$ (where o is infinitely small) be involved; the first, second, third, &c. term, will be respectively as the flowing quantity ($A^{\frac{m}{n}}$), the first, second, &c. fluxion of that quantity, as is demonstrated in the quadrature of curves, Sch. to Pr. 11. (for which see Harris's Lexicon.) Therefore $\frac{m^2 - mn}{2n^2} ooA^{\frac{m-2n}{n}}$ is as the second fluxion of $A^{\frac{m}{n}}$. Now if a (centrifugal) force act from the line CD in the direction B, and a body move in the curve CZ, the fluxion of A will continue the same always, and the force acting upon the body will be every where as the second fluxion of B, or of

Fig.

of $A^{\frac{m}{n}}$ (its equal), and therefore as $\frac{m^2-mn}{2n^2}$ $ooA^{\frac{m-2n}{n}}$, or (becaufe o is given,) as $\frac{m^2-mn}{n2}A^{\frac{m-2n}{n}}$, or as $\frac{m^2-mn}{n^2}B^{\frac{m-2n}{m}}$. Or $\frac{mn-m^2}{n^2}B^{\frac{m-2n}{n}}$ is as the centripetal force. This may be eafilier fhewn by fluxions only, by finding the 2d fluxion of (B or) $A^{\frac{m}{n}}$.

SECT. XIV.

34. [Pr. 94. equal to the fquare of HM] for imagine
35. a diameter drawn through H, and an ordinate to it through I, the abfciffa will be = MI, and the ordinate = MH. Alfo compleating the figure, the tangent IP cuts from the diameter H*a*, H*p* = H*u* = MI; therefore the triangles HLP, MLI are equal and fimilar, and HL = LM. And the latus rectum to H*a* is always the fame, whatever the angle of incidence be, if the velocity be given; for the line H*d*, that the body will defcribe in a given time, is given; and the line *dg*, that the body approaches the plane in a given time is given; and therefore $\frac{Ha}{Hz}$, or $\frac{\overline{zg}^2}{zH}$, or the latus rectum, is given.

[Pr. 96. Sch. of the fecants] of the angles between the line of incidence and the plane.

[Pr. 97, cor. 2.] The curve lines CP, CQ (being every where perpendicular to AP, DK,) are compofed of arches of circles; therefore PD is the increment of AP or AC, and QD the decrement of QK or CK; and therefore thefe increments are as the fines of incidence and emergence. And *è contra*, if PD, QD are as the fines of incidence and emergence, a body moving in the line PD, fhall emerge in the line QDK, by this Prop.

[Pr.

[Pr. 98. and therefore QS to be always equal to CE;] For since $qqqq$ and $ssss$ are always perpendicular to qs, therefore supposing QS and $1qs$ to intersect in K; then KQ — KS = K.$1q$ — KS, that is $qs = 1q.s$. Also let $1q$.K and $2q.s$ intersect in T, then $1q.s = 1q$.T — ST = 2T — sT = $2q.s$. Also $2q.s = 2q$ V — SV = $3q$.V — SV = $3q$.S; and so on, 'till at last qs, coinciding with CE, will be equal thereto. Therefore QS is every where equal to CE. Fig. 36.

[ib. Sch.] See the Author's Optics, where all these things are shewn.

BOOK

Fig.

BOOK II.

SECT. I.

[PR. 2. cor. For if that area] by Marq. Hoſpital's Conic Sect. art. 219. And my Conic Sect. Prop. 86, Hyperbola, cor. 6.

[ib. of the right line AC] by the converſe of Lem. 1; but the contrary does not hold, that is, if any area ABGD be taken for the time, that DC ſhall repreſent the velocity, or AD the ſpace.

[Pr. 3. And the reſiſtance] The force of gravity may be compared with the reſiſtance of the medium; for they may both be conſidered as uniform preſſure.

[ib. or AC to $\frac{1}{2}$AK] For ſuppoſe a tangent to be drawn to the point B, then Kq : (ſubtangent =) AC : : kq : Bk or AK. And in general in any area, as MstN; it will be, AC : $\frac{1}{2}$MN + NC : : (by the nature of the hyperbola) $\frac{Ms + Nt}{2}$: AB or Mm. And by diviſion AC : AM + $\frac{1}{2}$MN : : $\frac{Ms + Nt}{2}$: $\frac{ms + nt}{2}$: : area sMNt : area $smnt$. But AG : AM + $\frac{1}{2}$ MN : : force of gravity : reſiſtance in the middle of the fourth time. Therefore, sMNt : $smnt$: : force of gravity : reſiſtance in the midſt of the fourth time, and ſo of the reſt.

37. And the like demonſtration holds in aſcending motion; for let the rectangle ABDG be divided into innumerable rectangles Dk, Kl, Lm, Mn, &c. which ſhall be as the decrements of the velocities produced in ſo many equal times; then will AE, Ak, Al, Am, &c. be as the whole velocities, and therefore as the reſiſtances of the medium in the begin-

beginning of each of the equal times. Let AC : AK : : force of gravity : resistance in the beginning of the second time; then to the force of gravity add the resistances, and DEHC, K*k*HC, L*l*HC, M*m* HC, &c. will be as the absolute forces, or $\propto$ decrements of the velocities, $\propto$ D*k*, K*l*, L*m*, M*n*, &c. and therefore $\div$; whence DG*q*K $=$ K*qr*L $=$ L*rs*M $=$ M*st*N, &c. will be $\propto$ the equal times or forces. But AC : MC : : M*s* : AB. And by division AC : AM : : M*s* : *ms* : : area *s*MN*t* : area *smnt* : : (and therefore as) force of gravity : resistance in the fourth time; and so of the other areas. Therefore since DG*q*K, K*qr*L, L*rs*M, M*st*N, are $\propto$ the gravitating forces, the areas GE*kq*, *qklr*, *rlms*, *msnt*, $\propto$ resistances in each time, $\propto$ velocities $\propto$ spaces described. And by composition, in the times DG*r*L, DGBA, the spaces described will be as GE*lr*, GEB. *Q. E. D.* Fig. 37.

[ib. cor. 3.] For the diff. spaces $\propto$ velocities (that is by cor. 2.) $\propto$ CA, CK, CL ($\div$), or $\propto$ AK, KL, LM ($\div$) by lem. 1. (fig. 3.)

[ib. let that also be distinguish'd] This is true in this law of resistance; because the motion lost is $\propto$ velocity $\propto$ remaining velocity $\propto$ spaces described.

[Pr. 4. which is the locus of the point *r*. *Q. E. D.*] all the rest is plain by Prop. 2. and 3, and cor. Pr. 2.

[ib. cor. 1. that is, if the parallelogram] For DA : CP : : DR : RX $= \frac{DR \times CP}{DA}$. But DAB $=$ $\overline{DC - AC} \times AB =$ (by the nature of the hyperbola) $\overline{AB - AQ} \times DC = QB \times DC =$ (by construction) N $\times$ CP. And DA $= \frac{N \times CP}{AB}$. Wherefore RX $= \frac{DR \times CP \times AB}{N \times CP} = \frac{DR \times AB}{N}$.

[ib.

Fig. [ib. cor. 4. D*ra*F is alſo given.] by this prop.

[ib. cor. 6.] for then 2DP $\propto \frac{\text{lat. rect.}}{\text{reſiſtance}} \propto \frac{\text{lat.}}{\text{velocity}} \propto \frac{\text{velocity}^2}{\text{velocity}} \propto$ velocity.

[ib. cor. 7. the ratio $\frac{Ff}{DF}$] For when this ratio is the ſame with the other, the curves deſcribed (and ſuppoſed to be deſcribed) in theſe two caſes, will be ſimilar.

SECT II.

Prop. 5. had been better expreſſed thus, (tho' the tranſlation agrees with the original.)

If a body is reſiſted in the duplicate ratio of its velocity, and moves by its innate force only, thro' a ſimilar medium; and the ſpaces be taken equal: I ſay, that the times of their deſcription are in a geometrical progreſſion increaſing; and that the velocities at the beginning of each of the times are in the ſame geometrical progreſſion (decreaſing or) inverſely.

For it is plain, that the times may be taken in ſuch a gometrical progreſſion, as that the ſpaces cannot be equal, nor the velocities in the ſame inverſe progreſſion. But if the ſpaces are equal, the times and velocities will be in a geometrical progreſſion, inverſe to one another. The demonſtration is the ſame however the propoſition be expreſſed. For it being proved that the lines AB, K*k*, L*l*, &c. being ſquared, the ſquares are as the differences of the lines; and alſo that the ſquares of the velocities are as the differences of the velocities. Therefore if AB and K*k* be taken as the velocities in the beginning of the times AK, KL; and an hyperbola be drawn thro' the points B, *k*, to the aſſymptote CD; (whoſe center let be C; all the other velocities will be as the lines L*l*, M*m*, &c.

becauſe

because the progression, of the lines as well as the velocities, continues the same all along. Whence the spaces (in these equal times) will be as the areas A*k*, K*l*, L*m*, &c. and in the time AM, the space will be as AM*m*B. Now conceive the area AM*m*B to be divided into the equal areas A*k*, K*l* L*m*, &c. Then will CA, CK, CL, &c. be ∺ increasing; and the parts (by Lem. 1.) AK, KL, LM, &c. which are as the times, will be in the same progression, increasing. Also the velocities AB, K*k*, L*l*, &c. (which are reciprocally as CA, CK, CL, &c.) will be in the same progression inversely. *Q. E. D.* Fig.

[Pr. 7, and times conjunctly] very small particles of time.

[ib. they will always describe spaces,] for the parts of space described in those several particles of time, are as the respective velocities × those several parts of time; that is, as the first velocity × each part of time. And by composition, the whole spaces are as the first velocities × whole times. *Q. E. D.*

[Prop. 7, cor. 3.] dele. Let those diameters applied to that power.

[Pr. 8.] in fig. 1, Pl. 2, *k* ought to be beyond *l* in respect of A, &c. because the resistance decreases. But the demonstration is the same any way. But this figure is not in the 1st or 2d Ed. of the original. In Prop. 8 and 9, AC represents the relative gravity, or its weight in the fluid.

[Pr. 9. cor. 7.] For the greatest velocity is given (by Pr. 8. cor. 2, 3.) And thence the time of acquiring that velocity in free space. And ABNK (AB*nk*) being given there is given AK, and AP (A*p*) and ATD (A*t*D). Then ADC : ADT (AD*t*) : : time of acquiring that greatest velocity in free space : time sought.

[Pr. 10. equal among themselves] and very small.

Fig. [ib. or $\frac{MI \times NI}{HI}$] as appears by ſimilar Δ's, produced by letting fall a ⊥ from N on HI.

If the velocity in GH be greater than the velocity in HI, the decrement is $\frac{GH}{T} - \frac{HI}{t}$, and ariſes from the reſiſtance and gravity together (becauſe gravity draws the body from the tangent into the arch HI); and if gravity act not, that decrement would be greater (for gravity accelerates the motion), by $\frac{2MI \times NI}{t \times HI}$; therefore the decrement, by the reſiſtance alone is $\frac{GH}{T} - \frac{HI}{t} + \frac{2MI \times NI}{t \times NI}$.

. [ib. will be NI;] for Q being a given (ratio or) quantity; Qo will always repreſent MN, whatever the magnitude of o be.

And the ordinate DI $=$ CH — MI. Alſo the value of MI in the ordinate EK is (becauſe o becomes $2.o$, and ſubſtituting $2 \times o$, for o,) $2Qo + 4Ro^2 + 8Ro^3 +$ &c. and therefore EK $=$ CH $- 2Qo - 4Ro^2 - 8So^3 -$ &c. Alſo in the ordinate BG, when o becomes $-o$, the value of MI (by ſubſtituting $-o$ for o) will be $-Qo + Ro^2 - So^3 +$ &c. And BG $=$ CH — MI (as before,) $= P + Qo - Ro^2 + So^3 -$ &c.

[ib. or $\frac{R + \frac{3}{2}So}{R}$] for (becauſe o is infinitely ſmall) R, $R + \frac{3}{2}So$, $R + 3So$, are both arithmetical and geometrical proportionals.

[ib. by ſubſtituting the values —] becomes

$$\overline{1 + \frac{3So}{2R} \times o\sqrt{1+QQ} - \frac{QRoo}{\sqrt{1+QQ}} - o\sqrt{1+QQ}}$$

—

Fig.

$$-\frac{QRoo}{\sqrt{1+QQ}}+\frac{\overline{2Qo+2Ro^2}\times\overline{Ro^2+So^3}}{o\sqrt{1+QQ}+\frac{QRoo}{\sqrt{1+QQ}}}=$$ (neglecting the superfluous powers of o, or the quantities infinitely less than the other,) $o\sqrt{1+QQ}-\frac{QRoo}{\sqrt{1+QQ}}+\frac{3So^3}{2R}\sqrt{1+QQ}-o\sqrt{1+QQ}-\frac{QRo^2}{\sqrt{1+QQ}}+\frac{2QRoo}{\sqrt{1+QQ}}=\frac{3Soo}{2R}\sqrt{1+QQ}.$

[ib. latus rectum $\frac{HN^2}{NI}$] or $\frac{HI^2}{NI}=$ (neglecting the superfluous powers of o) $\frac{oo\times\overline{1+QQ}}{Roo}=\frac{1+QQ}{R}$.

[ib. p. 33, that a body by ascending from P] or any other point of the quadrant PF, in the direction of the arch of that quadrant.

[ib. Ex. 3. the second term $\frac{m}{n}o-\frac{bb}{aa}o$] or rather $\frac{bb}{aa}o-\frac{m}{n}o=Qo$; but the square is the same either way.

[ib. Ex. 4. for Qo] $\frac{nbb}{A^{n+1}}o-\frac{d}{e}o=Qo$.

[ib. Sch. And therefore if a curve] for the quantity $\frac{S}{R^{\frac{4-n}{2}}}\div\frac{\overline{HT}|^{n-1}}{AC}$ is as the density, and if that be given the density is given.

[ib. than these hyperbola's here described.] For let the 2 mediums be of the same density in the vertex of the curves; then if the bodies be projected from A; the uniform medium being more dense, and the other less dense at A, the body moving in the uniform medium will be more impeded,

 and

Fig. and consequently will descend the more, than that describing the hyperbola, which is less resisted, and which therefore moves nearer a streight line. Therefore the body moving in the uniform medium is more distant from the assymptotes in the vertex of the figure than the other body. But if the densities of the mediums were supposed equal in A; then (by the same way of reasoning), the body in the uniform medium would be nearer the assymptotes in the vertex. And generally speaking, these curves cannot therefore differ very far.

[ib. will touch the hyperbola in G,] for suppose the tangent TG to intersect MX. Then (by fluxions) the distance between that point of intersection and V, is $= \frac{VX}{n}$. Then by sim. △'s, $\frac{VX}{n} : VG :: VX : TX - VG = nVG$. But $VY = nVG$. Therefore $TX = GY$.

[ib. and the velocity] for the velocity (when the gravity is given) is as the ordinate of the parabola, that is (the abscissa being given, by the gravity) as the $\sqrt{\text{latus rectum}}$.

[ib. Rule 1.] For the velocities are as the $\sqrt{}$'s of the latus rectum of two parabola's, that is, as $\sqrt{\frac{2XY^2}{nn+n \times VG}}$, that is, as $\sqrt{\frac{2AH^2}{AI}}$ to $\sqrt{\frac{2Ab^2}{AI}}$ which (by supposition) are in a ratio of equality, and therefore $\frac{AH}{AI} = \frac{Ab}{Ai}$. Also since the densities are equal; $\frac{n+2}{3XY}$ will be given, that is $\frac{n+2}{3AH} = \frac{n+2}{3Ab}$. Whence $AH = Ab$. Whence AH, AI, remain the same; and HX, which is composed of AI and the subtangent to the axis XH, is the same also.

[Rule

[Rule 2.] For velocity $\propto \sqrt{\frac{AH^2}{AI}} \propto$ (becaufe Fig.
AH is given) $\sqrt{\frac{1}{AI}}$.

[Rule 3. and therefore AH] for let b be the ratio, then $\frac{Ab}{Ai} = b\frac{AH}{AI}$. And $\frac{AH^2}{AI} = \frac{Ab^2}{Ai} = b\frac{AH}{AI} \times Ab$. Whence $Ab = \frac{AH}{b}$. Alfo $\frac{Ab}{Ai} = \frac{AH}{bAi} = b \times \frac{AH}{AI}$. Whence $Ai = \frac{AI}{bb}$.

[Rule 4. a little greater] fince AH decreafes fafter than GT, therefore the fum of all or every AH + GT is leffer than the firft AH + GT × number of them. And therefore the fum of all the denfities (being reciprocal thereto) is greater than the fum of as many denfities in A and G. But the mean denfity, is = fum of all the denfities ÷ by the number of them, and therefore is fomething greater than the $\frac{1}{2}$ fum of the denfities in A and G. But half fum denfities : denfity in A : : $\frac{AH + GT}{2}$; GT. Therefore mean denfity : denfity in A : : is in oratio a little greater than $\frac{1}{2} \times \overline{AH + GT}$: GT.

[ib. Rule 5.] for HX = AI + fubtangent = AI + nAI. XN being the axis

[ib. Rule 6. by how much] for the variation of curvature in one of thefe hyperbola's, where n is great, is leaft in the part AG, and fo it is in the curve of projection, where the velocity is greateft; fo they agree in that part. But they differ more in the part GK, where the projectile approaches to a uniform motion.

[ib. Rule 7.] AH to 2AI, ought to be 3AH to 4AI. (But the original is AH to 2AI.)

Fig. [ib. Rule 8. whose conjugate] hyperbola shall pass thro' the point C.

[ib. what has been said] for by supposing the index n to be negative, VG (which before was as VX^{-n}) will then be as VX^{n}, and the curve AG will be a parabola, and what was demonstrated generally of the hyperbola, will hold true (for any index, and therefore) for the parabola; and all things will follow as before substituting $-n$ for n.

38. But the computation may be made for the parabola, as for the other; thus, let $VG = bbXV^{n} = XC^{n}$. Then the subtangent $ZT = nVG$. And let $XC = a$, $ng = e$, $cC = o$. Then $VG = \overline{a+o}^{n} =$

$$a^{n} + na^{n-1}o + n\times\frac{n-1}{2}a^{n-2}oo + n\times\frac{n-1}{2}\times\frac{n-2}{3}a^{n-3}o^{3} = P - Qo - Roo - So^{3}.$$

Whence

$$Q = -na^{n-1}.\ R = -n\times\frac{n-1}{2}a^{n-2}.\ S = -n\times\frac{n-1}{2}\times\frac{n-2}{3}\times a^{n-3}.$$

Therefore the density in G is as $\left(\frac{S}{R\sqrt{1+QQ}} = \right)\frac{n-2}{3a\sqrt{1+n^{2}a^{2n-2}}}$, that is, as $\frac{1}{\sqrt{a^{2}+nna^{2n}}}$, or as $\frac{1}{\sqrt{a^{2}+nnee}}$, that is reciprocally as $\sqrt{XC^{2}+TZ^{2}}$. And the same may be found by fluxions, putting Q, R, S, for the 1st, 2d, 3d, fluxion of VG or XC^{n}.

SECT. III.

[Pr. 12, and inversly as the velocity;] for the time, of describing a given space, is reciprocally as the velocity.

[Pr. 13. case 1, 2, 3, the decrement or increment PQ] in a given time.

[ib.

[ib. cor. and triangle do.] therefore *ex equo*, &c. Fig.
[ib. Sch.] For draw the indefinite line BAP, 39.
and make BD perp. and equal to BA; and draw DF, AF parallel to BA, BD. Let AP be the velocity, $AP^2 + 2BAP$ the resistance, AB^2 the force of gravity. Draw DTP, cutting FA in T, and the time of the whole ascent will be as the triangle DTA.

For draw DVQ cutting off PQ the moment of velocity, and DTV the moment of the triangle; then the decrement of the velocity PQ will be as the resistance and gravity, $AP^2 + 2BAP + AB^2$; that is as BP^2. But the area DTV is to the area DPQ, as DT^2 to DP^2, or as DF^2 to BP^2; therefore since the area DPQ is as BP^2, the area DTV will be as the given quantity DF^2. Therefore the area ADT decreases uniformly as the time, by the subduction of given particles DTV; and therefore is proportional to the whole time of ascent.

[Pr. 14. From the area DET] This should be; from the moment KLON, subduct DTV or $m \times BD$ the moment of DET; and is only a false translation.

[ib. cor. or as V^2;] for the space is $\propto \square$ of the time, that is, $\propto$ the square of DET or of $BD \times M \propto$ (because $M = \frac{DE \times V}{DA}$) square of $\frac{BD \times DE \times V}{DA}$, that is (because BD, DE, DA are given) as V^2, or as $\frac{BD \times V^2}{AB}$.

[ib. Sch. instead of the uniform resistance made to an ascending body;] this differs from gravity, only in this; that it cannot generate any motion; but it acts after the same manner in all moving bodies in destroying their motion, as gravity does in destroying the motion of ascending bodies. Gravity acts uniformly in a given direction. The force

Fig. arising from tenacity acts uniformly, but always in a direction contrary to the motion of the body. And therefore when the body is at rest, it can induce no change in it. Now in the horizontal motion of a body in a fluid, which is resisted in part uniformly, one may substitute the force of gravity for that uniform resistance, as in Pr. 8, 9; 13, 14. And in the ascent or descent of the body in a fluid, instead of the force of gravity, one may substitute the sum or difference of that uniform force and the force of gravity, as in these propositions.

SECT. IV.

In Prop 15.

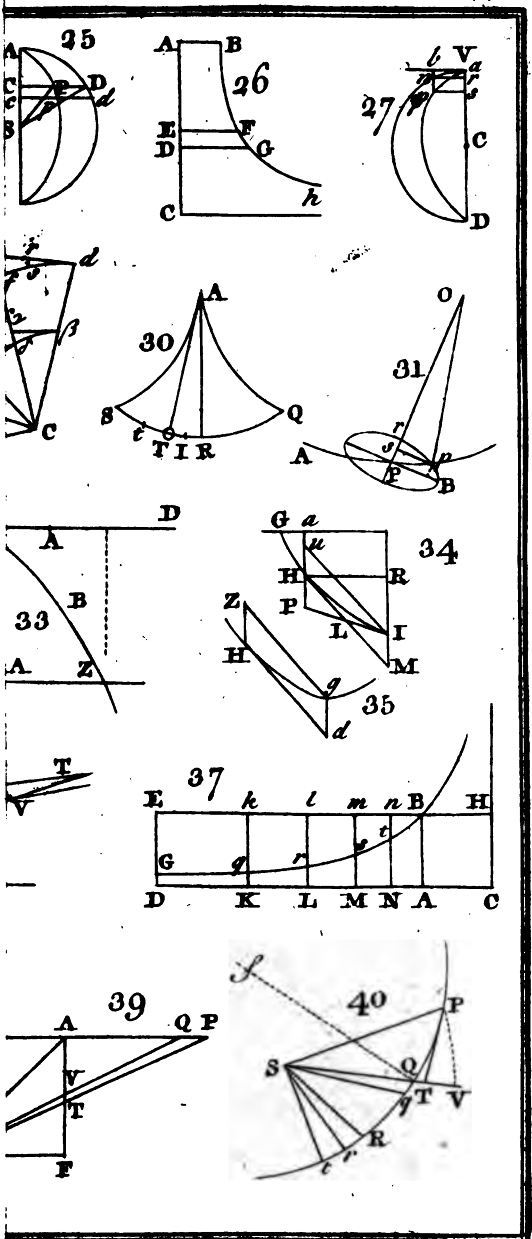
[And the decrements of these arcs arising from the resistance, will be as the squares of the times in which they are generated;] for since the arcs are infinitely small, the resistance affects the same as an uniform centripetal force would do.

40. [ib. The decrement of the arch PQ will be] for let q, t be the points it would arrive at, in the same times, in free space, then $Pq = qt$, and $PQ = Qr$, therefore $Qq = qt - Qr$; and adding Qq, $2Qq = Qt - Qr = tr$; and $Qq = \frac{1}{2}tr$: but $Qq = \frac{1}{4}Rt$; therefore $tr = \frac{1}{2}Rt = Rr$, and $Qq = \frac{1}{2}Rr$.

The same demonstration of this Prop. holds good, as well in ascending as in descending motion.

41. [Cor. 1. The velocity, &c.] for supposing PQ an arch of a circle whose center is f; then if PQ or PT be as the velocity in both curves, TQ will be as the centripetal force in both, but in both curves $TQ = \frac{PQ^2}{2Pf}$; therefore $Pf =$ rad. of the circle described with the same velocity.

[Cor.



Pl. III. pa 56.

[Cor. 4. the body will descend, &c.] for by cor. Fig. 1. the velocity in P in a resisting medium is = velocity in a circle at distance SP in free space; but velocity at distance SP in free space : to velocity at dist. $\frac{1}{2}$SP in free space (by cor. 6. Prop. 4. B. I.) or Prop. 34. : : as $\sqrt{\frac{1}{SP}}$: to $\sqrt{\frac{1}{\frac{1}{2}SP}}$: : that is, $\sqrt{1}$: $\sqrt{2}$; therefore, &c.

[Cor. 7, or as $\frac{2}{3}$AS to AB] (for this see my Geometrical Propotion, prop. 24.)

In Prop. 16.

[And the resistance, &c.] instead of this proportion in prop. 15, viz. $PQ : QR :: SQ : \sqrt{\overline{SP \times SQ}}$, substitute this $PQ : QR :: (SQ^{\frac{n}{2}} : SP^{\frac{n}{2}} ::)\ SQ : SQ + \frac{n}{2} SP - \frac{n}{2} SQ$, (by prop. 24, of my Geomet. Proportion) that is, as the describing velocities, &c.

[Cor. 1.] For the resistance : centripetal force : : $(\frac{1}{2}Rr =) \frac{\overline{1-\frac{1}{2}n} \times VQ \times PQ}{2SQ} : (TQ =) \frac{\frac{1}{2}PQ^2}{Sp} :: \overline{1-\frac{1}{2}n}\, VQ : PQ :: \overline{1-\frac{1}{2}n}\, OS : OP$.

SECT. V.

In Prop. 19.

[Case 3, different spherical parts have equal pressures,] that is, different spherical parts of equal magnitude.

In Prop. 20.

[And by a like reasoning, &c.] Supposing the thickness of the orbs to be reciprocally as the force of gravity or the density of the fluid, or in the complicate ratio of them; and it will appear (as before)

Fig. before) that the bottom is preſſed with a cylinder, of the ſame fluid; whoſe baſe is = the bottom, and height that of the fluid, by the ſame reaſoning as before, and prop. 19.

In Schol. Pr. 22.

[By a like reaſoning, &c.] Let the force of gravity be reciprocally as the nth power of the diſtance. The ſpecific gravities at A, B, C, &c. will $\propto \frac{AH}{SA^n}, \frac{BI}{SB^n}, \frac{CK}{SC^n}$, &c. and the denſities $\propto$ ſums of the preſſures $\propto \frac{AH \times AB}{SA^n}, \frac{BI \times BC}{SB^n}, \frac{CK \times CD}{SC^n}$, &c. $\propto \frac{AH}{SA^{n-1}}, \frac{BI}{SB^{n-1}}, \frac{CK}{SC^{n-1}}$, &c. And tu, uw, &c. $\propto$ differences of the denſities $\propto \frac{AH}{SA^{n-1}}, \frac{BI}{SB^{n-1}}$, &c. And $tu \times tb$, $uw \times ui \propto \frac{AH \times tb}{SA^{n-1}}, \frac{BI \times ui}{SB^{n-1}} \propto \frac{1}{SA^{n-1}}, \frac{1}{SB^{n-1}}, \propto \frac{1}{SA^{n-1}} - \frac{1}{SB^{n-1}}, \frac{1}{SB^{n-1}} - \frac{1}{SC^{n-1}}$, &c. and therefore univerſally $\frac{1}{SA^{n-1}} - \frac{1}{SE^{n-1}} \propto$ hyperbolic area $tbmy$. Therefore if $\frac{SA^n}{SA^{n-1}}, \frac{SA^n}{SB^{n-1}}, \frac{SA^n}{SC^{n-1}}$, or $\frac{1}{SA^{n-1}}, \frac{1}{SB^{n-1}}, \frac{1}{SC^{n-1}}$, be $\div$; then $\frac{1}{SA^{n-1}} - \frac{1}{SB^{n-1}} = \frac{1}{SB^{n-1}} - \frac{1}{SC^{n-1}}$, and the areas proportional thereto will be equal, viz. $thiu = uikw$, and therefore St, Su, Sw, that is, AH, BI, CK $\because$. *Q. E. D.* then putting n ſucceſſively = 3, 4, 0 — 1, &c. the truth of the ſaid Scholium will appear. And ſo on, *ad infinitum*.

[Other

[Other laws of condensation, &c.] Suppose the compressing force ∝ rth power of the density. Then the specific gravities in A, B, &c. ∝ $\frac{AH}{SA^2}$, $\frac{BI}{SB^2}$, &c. And the pressures in A, B, &c. ∝ $\frac{AH}{SA} + \frac{BI}{SB} + \frac{CK}{SC} + \frac{DL}{SD}$ &c, $\frac{BI}{SB} + \frac{CK}{SC} + \frac{DL}{SD}$, &c. Now if these sums were ∺, their differences $\frac{AH}{SA}$, $\frac{BI}{SB}$, &c. would be in the same ∺, and then the pressures in A, B, &c. ∝ $\frac{AH}{SA}$, $\frac{BI}{SB}$, &c. ∝ $\overline{\text{densities}}^{\,r}$ ∝ AH^r, BI^r, &c. And therefore AH^{r-1}, BI^{r-1} ∝ $\frac{1}{SA}$, $\frac{1}{SB}$. And AH, BI ∝ $\frac{1}{SA^{\frac{1}{r-1}}}$, $\frac{1}{SB^{\frac{1}{r-1}}}$. Then putting $r = \frac{4}{3}$, $\frac{5}{3}$, 2, &c. all things will appear as in this Schol. But now the aforesaid sums are not in geometrical progression, because it is plain their differences are not, viz. $\frac{AH}{SA}$, $\frac{BI}{SB}$, &c. for if $\frac{AH}{SA}$, $\frac{BI}{SB}$, &c. ∺, and seeing SA, SB &c. are ∺, therefore (by Prop. 18. cor. 2. of my Geom. Proportion) AH, BI, &c. would be ∺ which is false (except in the case of Prop. 21.) therefore neither these differences, nor sums are ∺. And consequently the conclusions of this Schol. in this respect are not true. And therefore it must be presumed, that the Author meant them only to be nearly, and not perfectly true.

Fig.

In Schol. Pr. 23.

[All theſe things are, &c.] It ſeems to me that the prop. holds true, tho' the centrifugal force be extended to all diſtances, in any given law, as well as when it ends at the next particle in the ſame given law. For ſuppoſing the particles of the fluid to be placed in parallel planes, and that theſe planes act on each other; the force of compreſſion on any one plane will be made up of the ſeveral forces, of all the other equidiſtant parallel planes. Therefore ſince the diſtances of all theſe planes before compreſſion are reſpectively proportional to their diſtances after compreſſion from this given plane; their actions on the given plane before and after compreſſion will alſo be proportional, (ſeeing their actions or forces are as ſome power of the diſtances); and their ſums will be alſo proportional to thoſe of any two correſponding planes, viz. as the neareſt to the neareſt. In theſe ſort of particles there will be required a greater force to produce an equal condenſation of an equal quantity of the fluid. But the denſity will be proportional to the compreſſion, as before.

SECT VI.

In Prop. 24.

Cor. 5. —— for let m, M = quantities of matter, w, W = weights, t, T = times, for the common pendulum l. Alſo L = new pendulum. T = new time, for M, and W. Then m : M : : wtt : WTT (by the Prop.) : : $\frac{wtt}{l}$: ($\frac{wtt}{l}$ =) $\frac{\text{WTT}}{\text{L}}$. And if m, M; t, T, are equal; $w \propto l$, which is cor. 4.

In

In Prop. 27.

[If the refiftance in the arc B, &c.] If the refiftance in the arc A were to the refiftance in the arc B as AA to AB, the times would be equal, and therefore refiftance AA in the arc A caufes the excefs of time above that in a non-refifting medium; and refiftance AB in the arc B caufes the excefs of time in B above that in a non-refifting medium, equal to the former excefs, (becaufe with thefe refiftances) they are defcribed in equal time. But now if B be defcribed with a refiftance BB; the excefs of time will be caus'd by that refiftance.

[But thofe exceffes, &c.] Now the exceffes of time caufed by defcribing B with the refiftances AB, BB refpectively, are as the fpaces left undefcribed after the given time; that is, as the refiftances AB, BB nearly, or as A to B, viz. as the arches A, B. And fince the excefs of time by defcribing A with refiftance AA = excefs of time by defcribing B with refiftance AB. Therefore it follows, that excefs of time in A : to excefs of time in B : : as A : to B nearly.

In Prop. 29.

[Area PIEQ may be to the hyperbolic area PITS : : as BC to C*a*; and that the area IEF may be to the area ILT as OQ to OS.]

That this is poffible (affuming P at pleafure) may be thus fhewn. Let the areas PIEQ, PITS be taken (as BC to C*a*) indefinitely fmall, then PIEQ muft be greater or at leaft equal to PITS; fuppofe them equal, then will PIFQ > PILS, and IF > IL (and more fo, if PIEQ > PITS), but when S, P, Q coincide OS to OQ is a ratio of equality, therefore in this cafe IFE to ILT (or IF to IL) > OQ to OS (or however not lefs), increafe the areas PIEQ and PITS in the fame ratio, and the

Fig. the ratio of OQ to OS will also increase and converge at length to the ratio of IEF to ILT; which shews the possibility of what was required; (but as to an actual solution I shall refer it to an algebraic process.) This being done as required; then, the points O, S, P, Q are all fixt, and the point R only variable.

[And the increment, &c.] for, the fluxion of $\frac{IEF}{OQ}$ OR — IGH to the fluxion of — PIGR : : as $\frac{IEF}{OQ}$ — HG to — RG : : or as HG — $\frac{IEF}{OQ}$ to RG.

Or rather thus, the increment of $\frac{IEF}{OQ}$ OR—IGH, is as $\frac{IET}{OQ} - HG \times - Rr = HG \times Rr - \frac{IEF}{OQ} \times$ Rr. And the increment of PIGR is as RG × — Rr; and therefore the decrement (because the decrement is a negative increment) of PIGR, is as RG × Rr.

In cor. to Pr. 30.

[Now if the resistance DK be in the duplicate, &c.] for the velocities being as the ordinates of a circle or ellipsis described on *a*B; the resistances (being as the squares of these velocities) are as the squares of these ordinates, that is, as the rectangles of the abscissa's. Therefore the resistances DK, OV are respectively as *a*DB, (*a*OB =) OB^2. But this is the property of the parabola *a*VB described to the axis VO, by cor. 2. to p. 4. of the parabola.

Gen. Sch. Pr. 31, pa. 96.

[But in lesser oscillations somewhat greater, &c.] that is, the difference in the lesser arc to the difference in the greater arc, is in a greater ratio than the squares of these lesser and greater arcs.

[pa.

[pa. 97.] for $\frac{1}{2}$ the cords of theſe arcs, read the Fig. cords of thoſe half arcs.

[ib. p. 98. As $\frac{7}{11}AV + \frac{7}{10}BV^{\frac{3}{2}} + \frac{3}{4}CV^{2}$.]

Theſe three members are taken to denote three laws of reſiſtance, viz. the firſt in the ſimple ratio of the velocity, the third in the duplicate ratio thereof, and the ſecond partly in the ſimple partly in the duplicate ratio (or in a mean between them); and $\frac{7}{10}$ the coefficient thereof is a middle one between $\frac{7}{11}$ and $\frac{3}{4}$.

[As 0,041748 to 121] it ſhould be as 0,041778 to 121.

[pa. 99. It is manifeſt that the force, &c.] for the forces of reſiſtance and gravity are as their effects, to wit, as the velocity loſt (by reſiſtance) and velocity gained (by gravity) in the ſame time, as 1 to $376\frac{1}{50}$.

[I alſo counted——I leave the calculation, &c.] the calculation after the preceding manner will be as follows.

Mean oſcillations	—	$3\frac{1}{2}$	7	14	28	56	112
diff. between 1ſt deſ. and laſt aſ.		$\frac{1}{2}$	1	2	4	8	16
diff. of arcs deſcribed in the deſcent and ſubſequent aſcent.		$\frac{1}{748}$	$\frac{1}{272}$	$\frac{4}{325}$	$\frac{12}{250}$	$\frac{24}{125}$	$\frac{48}{68}$

Alſo theſe in the greater arches are nearly in the duplicate ratio of the velocities, but in the leſſer arches ſomewhat greater, (theſe arcs are in the laſt ſeries of the table).

Let V = velocity in 2d, 4th, and 6th caſe = 1, 4, 16 reſpectively. And the difference of the arcs will be, in the

2d Caſe $= \frac{1}{272} = A + B + C$

4th Caſe $= \frac{12}{250} = \frac{6}{125} = 4A + 8B + 16C$

6th Caſe $= \frac{48}{68} = \frac{12}{17} = 16A + 64B + 256C$.

whence A = ,0005098; B = ,0005882; C = ,0025784. Therefore the difference of the

arcs

Fig. arcs is as $,0005098V + ,0005882V^{\frac{3}{2}} + ,0025784V^2$. And consequently the resistance of the globe in the middle of the arc, will be to its weight (by cor. pr. 30.) as $,00032442V + ,00041174V^{\frac{3}{2}} + ,0019338V^2$, to 121. Therefore the resistance will be to the weight, in case 2d, 4th, and 6th, as ,00267 to 121; ,0355324 to 121; ,5265949 to 121, respectively.

Note, you may take any other numbers (that are in the same proportion with these above) for V; for example I took $\frac{1}{4}$, 1, and $4 = V$, in the 2d, 4th and 6th cases, and A will be $= ,002039$; $B = ,004706$; $C = ,041255$; and the resistance to the weight (at the last) comes out the same as before.

In the 6th case the point marked in the thread described an arc of $112 - \frac{48}{68}$ inches $= 111\frac{5}{17}$; therefore the center of the globe describes an arc $= 115\frac{167}{187} = 115\frac{8}{9}$ nearly; and its velocity is (nearly) the same as in descending an arc $57\frac{17}{18}$ of a cycloid (whose semi-arc is 126,) or = velocity acquired by falling perpendicularly thro' the versed sine (or abscissa) of that arc $57\frac{17}{18}$, but this versed sine is $= 13,324$. Therefore the velocity of the pendulum is = velocity acquired by falling perpendicularly 13,324 inches. And with this velocity, the globe meets with a resistance which is to its weight as ,52659 to 121, or (taking that part only of the resistance which is as the square of the velocity,) as ,49505 to 121.

Also if a globe of water of equal magnitude moves with the same velocity, its resistance will be to its weight as ,49505 to 213,4; or as 1 to $431\frac{1}{15}$. Whence, in the time in which the globe uniformly describes 26,648 inches, the weight of the globe of water will generate all that velocity in the falling globe; therefore the velocity destroyed

by

by resistance, will be to that acquired by gravity (in the same time), as 1 to $431\frac{1}{11}$, or velocity lost $= \frac{1}{431\frac{1}{11}}$ of the whole velocity. And therefore in the time it would uniformly describe its semidiameter, with the same velocity, it would lose the $\frac{1}{3341.7}$ part of its motion. Fig.

[pa. 101,2, after five oscillations] this certainly should be after 10 oscillations, as appears by the process of the calculation.

[pa. 102. So that the difference 0,4475, &c.] for the motion lost is $\propto$ the resistance, and that $\propto$ the square of the velocity and square of the time nearly, which in this case is a constant product.

[pa. 103, l. 6.] Here is a small error, viz. ,61675 instead of ,61705; whence 45,453 would have been 45,43; and the resistances as 7,002 to 1; but this is not material.

[pa. 104. because the resistance, &c.] for the resistance $\propto$ square of the time $\times$ square of the velocity, which product is invariable, because equal arches are always described.

SECT. VII.

[Pr. 32. ratio of density] he means any given ratio of density, which are proportional to the particles.

[Pr. 32. Proportional times] any times, in that constant ratio, that is, let them move among one another, in similar directions, with velocities which are as the particles.

[Pr. 32. cor. 2. spaces proportional to their diameters] in these proportional times.

[Pr. 33. cor. 2. for if the forces, &c.] for the resistance arises from the centrifugal forces, and from the collisions, of the particles. The resistances of the first sort are as the motive forces,

E that

Fig. that is, as the accelerative forces and quantity of matter, that is, (by ſuppoſition) as the ſquares of the velocities and quantities of matter, that is, (becauſe the quantities of matter are given, the fluid being the ſame) as the ſquares of the velocities.

The reſiſtances of the ſecond ſort are as the number of reflexions, and their forces; that is, (as is proved in the Prop.) as the ſquares of the velocities, ſquares of the diameters, and denſities of the parts, that is (becauſe the diameters and denſities are given) as the ſquares of the velocities, accurately.

[As alſo the bodies E and G;] let E and G be vaſtly ſwifter than D and F.

[Pr. 34. but the former of, &c.] for $bH \times CB =$ ($BE^2 =$) ObN, which is the property of the parabola.

[Pr. 34. Schol. leſs reſiſted than the former ſolid;] for, if FG and IH be produced till they interſect, they will form a right angle; and therefore the fruſtum FGBHI will be leſs reſiſted than if the lines ſhould interſect in an obtuſe angle, (as is ſhewn in the cone;) which they would do if the lines fell within FG and IH; and for the ſame reaſon the reſiſtance would be greater in the curve itſelf, becauſe the lineolæ which conſtitute it, will, when produced, interſect one another at obtuſe angles. What is mentioned beſides in this Schol. is demonſtrated in the appendix.

[Pr. 35. it follows, that the cylinder, &c.] for mot. cylinder : mot. medium : : cylinder's magnitude $\times$ denſity $\times$ velocity : (medium's mag. $\times$ denſ. $\times$ vel. $=$) cylinder's $\frac{\text{magn.}}{2} \times 2$ vel. $\times$ medium's denſity : : cylinder's denſity : medium's denſity.

[ib. to the force by which] to the motive force by which its whole motion, &c.

[Pr. 35. cor. 2.] for, the reſiſtance of the globe and the force (that will quite take away its motion in the time it moves two or four thirds of its diameter,)

ameter,) are in a given ratio; therefore increase Fig. the velocity of the globe in any ratio, and the force (that will utterly deſtroy all its motion in a given ſpace) will be increaſed in a ratio of the velocity directly and the time inverſely, that is, as the ſquare of the velocity, therefore the reſiſtance of the globe is increaſed in the duplicate ratio of the velocity.

[ib. cor. 3.] for the ſame reaſon, increaſe the diameter of the globe in any ratio, and the motive force (that will utterly deſtroy its motion) will be increaſed in the triplicate ratio of the diameter directly, and the time inverſely, that is, in the triplicate ratio of the diameter directly, and the ſimple ratio of the diameter inverſely, that is, as the ſquare of the diameter; and therefore the reſiſtance of the globe will be encreaſed in the duplicate ratio of the diameter.

[Cor. 6. and its reſiſtance, &c.] Let BC be the reſiſtance at firſt, then reſiſtance at the firſt : reſiſtance at the end : : $BC^2 : EF^2$; but AE : AB : : BC : EF : : EF : BH; therefore $BC^2 : EF^2$: : BC : BH. $\therefore$ BH is the reſiſtance at the end.

[Cor. 7. As the logarithm, &c.] For (by Schol. 42. Prop. 86, hyperbola,) it appears, that if $AH = 1$. $AB = T$. $BK = t$: then it will be CBKF : EHKF : :

$$(\log. T+t - \log. T =) \log. \frac{T+t}{T} : \log. \overline{T+t}.$$

And EHKF : AHED : : ($\log. \overline{T+t}$: ,434?9448 : :) $\log. \overline{T+t} \times 2{,}30258509 : 1$. Alſo AHED : CBKG : : HE : $CB \times t$: : T : t. Therefore *ex equo*,

$$\text{CBKF} : \text{CBKG} : : \log. \frac{\overline{T+t}}{T} \times 2{,}30258 \times T : t : :$$

$$\log. \frac{\overline{T+t}}{T} \times 2{,}30258509 : \frac{t}{T}.$$

[Pr. 36. cor. 2.] Forces are equal when their effects are equal in a given time; but the effect of the motion of the effluent water (in the time a bo-

 dy

Fig. dy descends thro' GI) = a cylinder, whose length is 2GI, and base the hole EF; also the effect of the gravity or weight of a column whose height is GI, and base the hole EF = (by reason of an accelerated motion, and in the same time) a cylinder whose length is GI, and base the hole EF; and therefore the effect of the pressure of twice that column (in the same time) = a cylinder, whose length is 2GI, and base the hole EF; = the first cylinder, *ergo*, &c.

[Cor. 3. or IH + IO to 2IH.] for *ib* : *bo* : : *io* : *og*; or *bo* : *og* : : *ib* : *io*. And *bo* + *og* : 2*bo* : : *ib* + *io* : 2*ib* : : *io* + *ig* : 2*io* : : ⊙EF + ⊙AB : 2 ⊙EF.

[ib. cor. 10.] This cor. ought to be more exactly computed and demonstrated, because the following propositions depend thereon.

[Pr. 37. pa. 137. And is therefore nearly equal, &c.] This resistance = weight of a cylinder (whose base is that little circle and altitude $\frac{1}{2}$IG, from which altitude the cylinder must fall, &c.) = force by which its motion may be generated, &c.

[Pr. 37. cor. 1.] Let F = force (mentioned in the prop.) M = medium's density. C = cylinder's density. V = velocity. D = diameter. T = time (of describing four times its length). R = Resistance. Then $R = F \frac{\times M}{C}$ (by the prop.) $\propto C \times$

$$\frac{V}{T} \times D^2 \times \frac{M}{C} \propto V^2 D^2 M.$$

[Cor. 2.] Let EF be given, and density of the medium be also given. Then because this resistance = weight of a cylinder (whose base is PQ, and height $\frac{1}{2}$IG) $\times \frac{EF^2}{EF^2 - \frac{1}{2}PQ^2}$; therefore weight of this cylinder = resistance $\times \frac{EF^2 - \frac{1}{2}PQ^2}{EF^2}$. But the force requisite to take away the motion of this cylinder,

cylinder, whilst it moves four times its diameter Fig.
$\propto$ weight of this cylinder $\times$ velocity $\times$ $\frac{1}{\text{time of describing 4 times its length}}$ $\propto$ resistance $\times \frac{EF^2 - \frac{1}{2}PQ^2}{EF^2} \times \frac{EF^2 - PQ^2}{EF^2} \times \frac{EF^2 - PQ^2}{EF^2}$. And the density of the cylinder being increased, the force is increased in the same ratio.

[Cor. 3.] for since res. : force : : EF^6 $\times$ density medium : $\overline{EF^2 - \frac{1}{2}PQ^2} \times \overline{EF^2 - PQ^2}^2$ $\times$ density cylinder : : med. density : $\frac{\overline{EF^2 - \frac{1}{2}PQ^2} \times \overline{EF^2 - PQ^2}^2}{EF^6}$ $\times$ cyl. density; increase the distance it moves, in any ratio; and the force (to destroy the motion) will be decreased in that ratio, as suppose the ratio of $\frac{\overline{EF^2 - \frac{1}{2}PQ^2} \times \overline{EF^2 - PQ^2}^2}{EF^6}$. Then res. : force : : med. density : cylin. density.

[Sch. pr. 37. AE and BE be 2 parabolic arcs] 43.
since the resistance is the same, whether the fluid or the cylinder move, therefore the case of the resistance of the circle in fig. 4. is the same case with the resistance of the circle AB in fig. 6, considering the velocities HG and EG, wherewith they are acted on; but (by cor. 9, pr. 36.) the resistance on PQ $= \frac{1}{2}$ cylinder on PQ and height GH (thro' which a falling body acquires the velocity, which the fluid has on PQ), but $\frac{1}{2}$ this cylinder is $=$ a parabolic spindle on PQ and height GH, nearly. Therefore the resistance on AB $= \frac{1}{2}$ cylinder on AB and height GE ($=$ GH,) thro' which a falling body acquires the velocity with which the cylinder moves,) which is equal to a parabolic spindle on AB and height GE $=$ AEB by construction, (since lat. rect : (HG $=$) GE : : GE : AG.) This is the very construction of fig. 2.

Fig. [ib. must be to this force as 2 to 3, at least,] for the weight upon the little circle (in cor. 7. p. 36.) is there shewn to be equal to a cylinder of $\frac{1}{3}$ the height GH at least. But the weight of a cylinder on that circle and $\frac{1}{2}$ that height is the force whereby the cylinder's motion may be generated in the time it moves 4 times its length or 2GH; therefore the resistance in that cor. is to the weight here demonstrated (*ceteris paribus*) that is, to the force : : as $\frac{1}{3}$ to $\frac{1}{2}$: : that is as 2 to 3, at least; and increasing the density, the force is increased in the same ratio.

[ib. let CF and DF.] It is necessary to allow as much time for the meeting of the particles of water (at the axis of the solid from the side thereof), after they are past the solid, as for their separation before they come to it. Now suppose this to be the case of fig. 4. In the time that a particle passes from H to P, the same particle with the velocity in P, will in the same time describe 2HG, and therefore must pass twice as far to arrive to the axis again. Whence in fig. 6, the particle F must be twice as far from CD, as E from AB; but the height of FCD = 2 height of AEB, by construction. For let z, y = 2 ordinates to $x = \frac{1}{2}$AB; and $zz = 4rx = 4ry$; and $z = 2y$.

[Cor. 1. Pr. 38.] the demonstration is the same with that of cor. 1. pr. 37.

[Cor. 2, Pr. 38.] Let density of globe : dens. of fluid : : d : 1. Then $d \times \frac{4}{3}$ diameter of the globe = space fallen, and $d \times \frac{8}{3}$ diam. globe = space it describes afterwards, in the time of its fall. Then, because forces (which generate a given motion), are reciprocally as the (times, that is reciprocally as the) spaces which any body describes with the same uniform velocity. Therefore, the force of the globe's comparative weight, (which generates its motion, in the time it describes $d \times \frac{8}{3}$ diam.) : force that generates the same motion, (while it describes $\frac{8}{3}$ diam.)

$\frac{8}{3}$ diam.) : : $\frac{8}{3}$ diam. : $d \times \frac{8}{3}$ diam. : : 1 : d : : density of the fluid : density of the globe. Therefore, &c. Fig.

[Cor. 3. Pr. 38.] The velocity being given, the motion M is also given; but (by this Prop.) resistance $\times$ d = force, which will generate the motion M (in the time of moving $\frac{8}{3}$ diameter); but this force must be = resistance, which will quite take away the said motion M (in the same time of moving $\frac{8}{3}$ diameter). Now augment the time of moving (preserving the velocity) in any ratio, (let it = T.) And this last resistance (which will quite take away the said motion M, in the time T) will be decreased in the same ratio (= R). Therefore T and R are also given. Therefore by cor. 7, &c.

[Cor. 4. Prop. 38.] for let M = $\frac{8}{3}$ diameter in the time T; and because (by sup.) $\frac{tM}{T+t} = \frac{1}{2}M$, therefore T = t. Now (by cor. 7. pr. 35.) it will be $\left(\frac{t}{T}=\right)$ 1 : (log. $\frac{T+t}{T} \times 2, 3$, &c. = log. 2×2, 3, &c. =) ,693 : : $\frac{8}{3}$ diameter : $\frac{5{\cdot}545}{3}$ diameters, which is less than two diameters. Now that M ought to be = $\frac{8}{3}$ diameter in the time T is plain; for it is required (by cor. 7. pr. 35.) that the resistance R quite takes away M in the time T. But (by this prop. 38.) the resistance (= force required) will just take away the motion M in moving $\frac{8}{3}$ diameter. But if T be any other time than that of moving $\frac{8}{3}$ diameter, then the force to destroy M in that time will not be = resistance, but greater or lesser than it.

[Prop. 40. let the globe, &c.] This is calculated in Prob. XIV. of my book of Fluxions. In cor. 2. of that Prob. G, H and N are found, and thence

Fig. the velocity $= \frac{N-1}{N+1}$ H, where H is the greateſt velocity the globe can acquire, in falling in the fluid; and the height fallen comes out the ſame in value with his, but in other ſymbols; but the one may be eaſily reduced to the other.

[ib. the numbers in the fourth column, &c.] 2F is the ſpace deſcribed in the time G with the greateſt velocity H, (by the Prob.) therefore, G : 2F : : P : $\frac{2P}{G}$ F = the ſpace deſcribed in the time P with the greateſt velocity, the numbers in the fourth column.

[Sch. Pr. 40. and any other globe, &c.] that is, the magnitude of any body $\propto$ the exceſs of its weight in vacuo above that in water. Or 132,8 : 1 : : this ſaid exceſs: magnitude of the globe. Hence, Diameter globe (in Exp. 1.) = 0,84224.

[ib. Exp. 1. but this ſpace, by reaſon, &c.] to demonſtrate this, it is neceſſary to premiſe, that, the greateſt velocity wherewith a globe can deſcend by its comparative weight thro' a fluid in a canal, is that which it may acquire by falling with the ſame weight, without reſiſtance, and in its fall deſcribing a ſpace which is to $\frac{4}{3}$ diameter as mn^2d, to l^3. Puting denſity globe : denſity fluid : : d : 1. and l = orifice of the veſſel. N = l — great circle of the globe, $m = l - \frac{1}{2}$ great circle. For (as in cor. 2. Pr. 38.) the ſpace it afterwards deſcribes (in the ſame time of its fall) $= \frac{mn^2d}{l^3} \times \frac{8}{3}$ diameter; then the force of the globe's comparative weight (which generates its motion, in the time it deſcribes $\frac{mn^2d}{l^3} \times \frac{8}{3}$ diameter) : force (that generates the ſame motion in the time it deſcribes $\frac{8}{3}$ diameter) : : ($\frac{8}{3}$ dia-

($\frac{8}{3}$diameter : $\frac{mn^2d}{l^3}\times\frac{8}{3}$ diameter : :) $l^3 : mn^2d$; therefore (by Prop. 39.) the resistance of the globe = force of the globe's comparative weight (which generates this motion in the time it describes $\frac{mn^2d}{l^3}$ $\times\frac{8}{3}$ diameter, or falls $\frac{mn^2d}{l^3}\times\frac{4}{3}$ diam.) And therefore this force cannot accelerate the globe. *Q. E. D.* Fig.

Now,

By the experiment; $2F = d\times\frac{4}{3}$ diameter; but the space the globe uniformly describes (with the greatest velocity wherewith it can descend in the canal or vessel, and in the time of acquiring it with its compound weight) $= \frac{mn^2d}{l^3}\times\frac{8}{3}$ diameter $= \frac{mn^2}{l^3}\times 2F = \frac{mn^2}{l^3}\times 4.4251$. Also $\sqrt{95,219}$: $\sqrt{\frac{mn^2}{l^3}\times F}$: : $1''$: $\sqrt{\frac{mn^2}{l^3}}\times\sqrt{\frac{F}{95,219}} = \sqrt{\frac{mn^2}{l^3}}$ $\times G = \sqrt{\frac{mn^2}{l^3}}\times 0,15244$ (by the laws of falling bodies) = time of acquiring the greatest velocity. And therefore ($\sqrt{\frac{mn^2}{l^3}}\times G$ (=time) : $\frac{mn^2}{l^3}\times 2F$ (= space) : :) $G : \sqrt{\frac{mn^2}{l^3}}\times 2F$: : $4''$: $\frac{n}{l}\sqrt{\frac{m}{l}}\times\frac{8F}{G}$ = (space described in $4''$) $= \frac{n}{l}\sqrt{\frac{m}{l}}\times 116,1245$. Subduct $\frac{mn^2}{l^3}\times 1.3862944F = \frac{mn^2}{l^3}\times 3,0676$. And $\frac{n}{l}\sqrt{\frac{m}{l}}\times 116,1245 - \frac{n}{l}\sqrt{\frac{m}{l}}\times 3,0676$ = space fallen in the fluid, = 116,1245

Fig. $-3{,}0676 \times \frac{n}{l}\sqrt{\frac{m}{l}}$, nearly, as the author makes it.

[ib. ex. 13.] This and the following are upon the ſame computation as the foregoing experiments in water, to wit, the theory of non-elaſtic fluids; the reaſon is, becauſe he conſiders fluids as elaſtic, whoſe parts are diſcontinued, &c. ſee Sch. Pr. 35.

[ib. pa. 160. equal to $\frac{1}{4586}$, &c.] This is plain from what follows, afterwards. For let a ſpace : $\frac{8}{3}D :: 860 : 1$; this ſpace $= \frac{6880}{3} D = 2293D$, and this ſpace the globe moves in the time T, with velocity V; let therefore $T = 2293$, and $t = \frac{1}{2}$. Then, motion loſt $= \frac{\frac{1}{2}V}{2293 + \frac{1}{2}} = \frac{1}{4587} V$; note $t\ (= \frac{1}{2})$ is not ſenſibly increaſed, by the reſiſtance, in moving thro' $\frac{1}{2}D$; and hence I have taken it for the ſame.

[ib. let D be the, &c.] for, put denſity globe : denſity fluid $:: d : 1$; alſo let the ſpace $\frac{8d}{3}D = S$. Now by Prop. 38, $d \times$ reſiſtance $=$ force, which (uniformly continued) will quite deſtroy its motion, in the time it moves $\frac{8}{3}D$; therefore $d \times$ reſiſt. will quite take away its motion, whilſt it moves $\frac{8}{3}D$. But (becauſe the force of that reſiſtance and the time of its acting are reciprocally proportional), the reſiſtance the globe meets with will quite take away its motion whilſt it moves (with V) $\frac{8d}{3}D = S$. Therefore (by cor. 7. Pr. 35.) the globe in the time T (of moving S), meets with a reſiſtance R, which will quite deſtroy its motion V; therefore in the time t, it will loſe $\frac{t}{T+t}V$, &c.

SECT.

Fig.

SECT VIII.

44.

[Pr. 42. and becaufe the motion of the waves, &c.] Let the waves A, B, C move from A towards C, and fuppofe the water to defcend from the vertex C into the hollow E, and thence up to B, which it will do fuppofing the vertices A, B, C to be of equal altitude; alfo let the water defcend from B to D and afcend to A, &c. Now fince the water in the vertex B defcends towards D, at the fame time that other water (that had defcended from C to E,) is afcending to B; therefore the water will be lower at the point B and higher at the point *b* than before; and confequently that the vertex of the wave B will be transferred towards C. Therefore the vertices (and hollows and any correfpondent points) of all the waves will move from thence towards C; therefore the waves will be carried on by the deflux of water down CE, BD; and the accumulation thereof, on EB, DA.

[Pr. 43. cafe 1. at nearly equal diftances, &c.] they fet forward at equal diftances, and continue fo, becaufe they move with equal velocity, as appears by (the demonft. of cafe 1, 2, 3 of) Prop. 48. 45.

[Pr. 46. becaufe the motion of the waves is carried on, &c.] Let A be the vertex of a wave. The fluid at *b* is more preffed (by reafon of a greater depth of incumbent water) than *d*; therefore that greater preffure caufes the water to recede from *b* in all directions, and to afcend at *c*; therefore the vertex of the wave is now in *c*. Alfo a greater preffure of water propagated from *d* to *f*, caufes the water to defcend at *c* and afcend at *e*, therefore the vertex of the wave moves along *a*, *c*, *e*, &c. And this it will always continue to do, if at firft any force is fuppofed to be applied

to

Fig. to the side *gba*, to hinder the ascent of the water there in like manner as it ascends at *c*, *e*; for then the side *gba* will be afterwards always descending, and the side *ace* ascending, without the continuance of such force.

[ib. transverse measure] is measuring along the surface of the water; as is plain from the demonstration.

46. [ib. cor. 2. rather performed in a circle] as in the note on Prop. 42. But if the breadth of the waves be measured, not in the right lines *acb*, but along the surface of the water (because the deflux of water thro' *b*, *c*, *a* is made in a curve line near a circle), then the time mentioned in the prop. will be very nearly as it is there assigned.

47. [Prop. 47,] fig. 1. pl. 9. is wrong, it should be thus; (and the original is wrong too,) where *l*, *m*, *n*, stand at N, M, L.

[ib.—with those of an oscillating pendulum.
ib. according to the law of a pendulum oscill.]
for, by Prop. 52. lib. 1. space described by a pendulum, in the time PI; is to a whole vibration; as the time PI, to the time PIS; and to two vibrations, as PI, to PHSP, and the accelerating force of a pendulum, is as its distance from the lowest point. See cor. Pr. 51.

[ib. that is, (because HL — KN is, &c.] that is, HL — KN : HK : : OM : OI.

[Pr. 48. case 1. distances of the pulses, &c.] the distances of the pulses in one medium be = distances of pulses in the other.

[ib. and therefore the pulses;] hence it appears that the correspondent parts in both mediums, (vibrate or) go and return together, or in equal times, tho' one makes longer vibrations than the other. But (by supposition) the distances of pulses in both mediums are equal; and the pulses in both

both are tranflated thro' thefe equal diftances in the times of the parts of the medium's (vibrating or) going and returning, which times are proved equal; therefore the pulfes in both mediums move equal diftances in equal times, and therefore are equally fwift. Fig.

[Pr. 48. cafe 2. then will their contractions, &c.] be equal, in equal fpaces.

[ib. and moreover, &c.] for fpace $\propto$ $\frac{\text{fquare of the time}}{\text{matter}}$ (for thefe fpaces are generated by accelerative forces), and time $\propto \sqrt{\text{fpace}} \times \sqrt{\text{matter}} \propto \sqrt{\text{fpace} \times \text{fpace}} \propto$ fpace.

[ib. Cafe 3. the time which is neceffary, &c.] for fpace $\propto \frac{\square \text{ time} \times \text{force}}{\text{matter}} \propto \frac{\square \text{ time} \times \text{force}}{\text{denfity}}$.

Therefore if the fpace be given, time $\propto \sqrt{\frac{\text{denfity}}{\text{force}}}$.

Therefore velocity $\propto \frac{1}{\text{time}} \propto \sqrt{\frac{\text{force}}{\text{denfity}}}$.

[Prop. 49. — ratio of PO to A conjunctly;] that is, time $\sqrt{\frac{\text{matter}}{\text{force}}} \propto \frac{1}{\sqrt{\text{force}}}$, when the quantity of matter is given, (this in accelerated motions). Then time of one vibration by that elaftic force : time of one vibration of a pendulum PO, by the weight : : $\sqrt{VV} : \sqrt{PO \times A}$, and time of one vibration of PO, by the weight : time of one vibration of A : : $\sqrt{PO} : \sqrt{A}$. Therefore, &c.

[ib. —of the going and returning] of the pendulum PO.

[ib. — of one ofcillation compofed of the going and returning,] of the pendulum A.

[ib.

Fig. [ib. cor. 1. — to its circumference.] For (Mechanics, Prop. 24.) diameter : circumference : : time of fall thro' $\frac{1}{2}$ radius : time of one vibration. Therefore as radius : circumference : : time of fall thro' $\frac{1}{2}$ radius : time of two vibrations.

[ib. cor. 2.] for velocity $\propto \frac{\text{space}}{\text{time}} \propto \frac{A}{\sqrt{A}} \propto$

$\sqrt{A} \propto \sqrt{\frac{e.\text{force}}{\text{density}}}$. By elastic force he means, that which arises by heat, as well as that arising from the compression.

[Prop. 50. Let the number] of the double vibrations, &c.

[ib. Schol. in the subduplicate ratio of the defect of the matter] that is, of the quantity of vapour or watery particles. And (by Prop. 4[illegible]or 2.) the velocity will be increased as (the [illegible]sity that is, as) the $\sqrt{}$quantity of true air in a given space, is decreased.

[ib. near 100 pulses in] This computation presupposes, that those vibrations of the string are double vibrations, or vibrations composed of its going and return.

SECT. IX.

[Prop. 51, but the differences of the angular motions, &c.] these are as the difference of the absolute motions round the axis, that is, as the difference of the absolute translations, or as the relative translations (which are those here spoken of) directly, and the distances inversely.

[ib. of quadratures of curves, &c.] Let SD $= x$; D$d = y$; then D$d \propto \frac{1}{xx}$ $\left(= \frac{b}{xx}\right)$ by construction; therefore $y = \frac{b}{xx}$; and $y\dot{x} = bx^{-2}\dot{x}$; and

bx

$\frac{bx^{-1}}{-1} = \frac{-b}{x} =$ area; therefore area $\propto \frac{1}{x} =$ DdQ.

[ib. cor. 3.] In a given time let the parts 1, 2, 3, 48.
of the fluid defcribe the fpaces $1r$, $2s$, $3t$; take away or add the equal angular motions $1n$, $2o$, $3p$; fo that the fpaces nr, os, pt be defcribed in that given time; now whether the points 1, 2, defcribe the fpaces $1r$, $2s$, or nr, os; the tranflations at the end of the given time will be $= si$; alfo if 2, 3 defcribe $2s$, $3t$, or os, pt, the tranflations at the end of the given time will be the fame in either cafe, viz. $= ut$; therefore the tranflations being equal and the attrition alfo, the motions will be continued.

[Pr. 52. cor. 8. about any given axis] paffing thro' the globe; the relative motion of the parts of the globe and fluid are the fame ftill, therefore, &c.

[ib. cor. 9. femidiameter of the globe;] let BC 49.
be a plane, D a point of the veffel, A a point of the globe; then period. time of A : time B : : $<$ velocity B : $<$ velocity A; and therefore time A + time B : time A : : (vel. B + vel. A =) $<$ vel. A from B : $<$ vel. B : : time B : time A from the plane BC (to BC again) : : CB^2 : CA^2. But time B from DC = time D from BC; therefore, time D (from BC) : time A (from BC) : : CB^2 : CA^2; as it fhould be by the prop. and cor. 8.

BOOK

BOOK III.

[PROP. 3. the action of the ſun, attracting, &c.] ſee cor. 7. Pr. 66. lib. 1, or cor. 14, where KL is $\propto$ PT. nearly, (SK, or ST being given); and the force TM in its mean quantity, is = PT, nearly.

[Prop. 4. and the ſpace which a heavy body deſcribes] by falling in 1″ : half the length of the pendulum (= $\frac{1}{2}$radius) : : ſquare of 1″ : ſquare of the time of falling thro' half the pendulum : : (Mechanics, Prop. 24.) ſquare of the circumference : ſquare of the diameter.

[ib. pa. 216. line 13, the mean diſtance of 60 diameters] it ſhould be 60 ſemidiameters, as it is in the original.

[Prop. 6. p. 222.—ſubduplicate of that proportion, as by ſome computations I have found,] *i. e.* as I found by making ſome calculation.

For let R be the diſtance of the Sun and Jupiter, d the greater force, e the leſſer; then to find the diſtance x where d may be diminiſhed to e. Becauſe the forces are reciprocally as the ſquares of the diſtances, ſay $\frac{1}{R^2} : \frac{1}{x^2} : : d : e : : x^2 : R^2$. And $x : R : : \sqrt{d} : \sqrt{e}$; therefore at the diſtance x, the weight d (of the ſatellite) would become equal to e; and the center of their orbits, would not be in Jupiter, contrary to experience. And if the force of the ſatellite was leſs; by a like computation, the center of their orbits would be nearer than Jupiter, alſo contrary to experience.

[Prop.

[Prop. 8. cor. 1. and $\frac{1}{169282}$ reſpectively] this Fig. ſhould be $\frac{1}{196282}$, or rather $\frac{1}{194075}$.

[ib. at the diſtances 10000, &c.] theſe are as the diameters; and the apparent diameters are found by aſtronomical obſervation; which × by the proportioned diſtances, gives the ratio of the real diameters.

[Pr. 8. cor. 4. — of ſo much the greater denſity.] This is not generally true, as appears by cor. 2 and 3.

[ib. cor. 3. — truely defined.] This is demonſtrated in Graveſand, L. IV. p. 232, 3. See Appendix, p. 20.

[Pr. 10. loſe almoſt a tenth part of its motion.] For by Schol. Pr. 40, p. 160, it is, $1 : 860 :: \frac{8}{3} : 2293.33 +$ = ſpace deſcribed in the time T; putting $D = 1$, $V = 1$; and $T : t :: 2293.33 : 2293.33 \frac{t}{T}$ = ſpace (uniformly) deſcribed in the time t. Then (pa. 161.) $\frac{t}{T} : 2{,}302$, &c. × Log. $\frac{T+t}{T} :: 2293.33 \frac{t}{T} : 229.5$; therefore $2293.33 \times 2.30258 \times \text{Log.} \frac{T+t}{T} = 229.5$; and $\log. \frac{T+t}{T} = {,}043461$; and therefore $\frac{T+t}{T} = 1.105+$; and $\frac{T}{T+t} = \frac{1}{1.105} = \frac{9}{10} +$ and $\frac{t}{T+t} = \frac{1}{10}$ nearly.

[ib. or as 75 (0^{12}) to 1 nearly.] In fig. 3. pl. 5. L. II. Let SA = rad. earth = 20949674 feet. AB = 1. AF = 200 miles = 1056000 feet; then, by cor. pr. 22. II. $Aa - Bb : Aa - Ff :: thin : thnz$; that is, $AB \times SF : AF \times SB ::$ or $1 : 1005325$

Fig. : : *thin* : *tbnz* : : (by sch. pr. 86 hyperbola) log. AH — log. BI : log. AH — log. FN; because water is 860 times denser than air, and the incumbent weight of the atmosphere at the surface of the earth is about = weight of 33 feet of water; therefore the weights (and consequently the densities) in A and B, are as 28380 and 28379; whence AH or S*t* = 28380, and BI or S*n* = 28379; and log. AH — log. BI = ,0000152, log. AH = 4,4530124; therefore 1 : 1005325 : : ,0000152 : 4,4530124 — log. FN; and log. FN = —11,1730. And therefore FN = ,000000000015, and density in F : density in A : : 15 : 28380000000000000 : : or as 1 : to 1800000000000000, which is less than 1 to 7500000000000; but the computation will vary as the radius of the earth, the density of water and air at the earth, is supposed to vary.

[ib. and hence the, &c.] as before (by Pr. 40. Sch.) since density of water : to density of air 200 miles from the earth : : 645 (0^{14}) : 1. Space described in time T = 172 (0^{15}). And $\frac{t}{T}$: log. $\frac{T+t}{T}$ × 2.302, &c. : : 172 (0^{15}) × $\frac{t}{T}$ (= space described in time *t*) : 2792250000 = space Jupiter describes in 1000000 years. Therefore log. $\frac{T+t}{T}$ = ,00000000705. And $\frac{T+t}{T}$ = 1.000000017. And $\frac{T}{T+t}$ = ,999999986. And $\frac{t}{T+t}$ = ,000000014, which is less than $\frac{1}{1000000}$.

[Pr. 13. are almost as 16, &c] for (by this and cor. 2. Pr. 8.) accelerative force of Jupiter towards Saturn :

Saturn : accelerative force of Jupiter towards the Sun : : $25 \times \frac{1}{3021}$: 16×1 : : 81 : $\frac{16 \times 81 \times 3021}{25}$. Fig.

[Pr. 14. Sch. And hence we may find, &c.] For by cor. 7. prop. 66. I. The apsides of their orbits move in consequentia. And by cor. 16, of the same, the motion of the apsides of the body P (plate 21. fig. 2.) will be as the periodical time of P directly, and the square of the periodical time of T inversely, that is, as the periodical time of P (because the periodic time of T, round S, or of the Sun round Jupiter or Saturn, or (which is the same thing) of Jupiter or Saturn round the Sun, is given); that is (by prop. 15. I.) in the sesquiplicate ratio of their distances PT.

[Pr. 19. in the duplicate proportion of the rad. 50.
to the cosine of latt.] For (by cor. 3. pr. 4. L. I.) centrifugal force at the equator : centrifugal force at a from the axis ($= ab$) : : rad. : cos. lat. : : $r : c$. Also the force directly from the axis : force directly from the earth : : $ab : be$: : $r : c$. Therefore, *ex equo*, centrifugal force at the equator : centrifugal force directly from the earth, at a : : $rr : cc$.

[ib. but by computation (from] Let GC $= t =$ 51.
101. BP $=$ C $=$ 100. PE $= x$. Then will $ED^2 = \frac{tt}{cc} \times \overline{cx - xx}$. And $ER^2 = PD^2 = \frac{ttx}{c} - \frac{tt}{cc}x^2 + x^2$. And $ER = \sqrt{\frac{tt}{c}x + \frac{cc - tt}{cc}x^2} =$ (suppose to) $\sqrt{r^2x - sx^2}$. Whence the fluxion of the area PQRE is $= \overline{r^2x - sx^2}|^{\frac{1}{2}}\dot{x} = rx^{\frac{1}{2}}\dot{x} - \frac{sx^3}{2r}\dot{x} - \frac{s^2x^{\frac{5}{2}}}{2.4r^3}\dot{x} - \frac{3s^3x^{\frac{7}{2}}}{2.4.6r^5}\dot{x} - \frac{3.5s^4x^{\frac{9}{2}}}{2.4.6.8r^7}\dot{x}$ — &c. And the area PQRE $= \frac{2}{3}rx^{\frac{3}{2}} - \frac{sx^{\frac{5}{2}}}{5r} - \frac{s^2x^{\frac{7}{2}}}{7.4r^3} - \frac{3s^3x^{\frac{9}{2}}}{9.4.6r^5}$

Fig. $-\frac{3.5.s^4x^{\frac{11}{2}}}{11.4.6.8r^7}$ — &c. $= \frac{2}{3}rx^{\frac{3}{2}} - \frac{3.sx}{2.5r^2}A + \frac{5.sx}{7.4r^2}B + \frac{7.3.sx}{9.6r^2}C + \frac{9.5.sx}{11.8.r^2}D$ + &c. = (putting $\frac{sx}{rr} = q$) $\frac{2}{3}rx^{\frac{3}{2}} - \frac{3}{10}Aq + \frac{5}{28}Bq + \frac{7}{18}Cq + \frac{45}{88}Dq$ + &c. = (when $x = c = 100$; and s being $= \frac{cc - tt}{cc} = ,0201$. And $r^2 = \frac{tt}{c} = 102,01$. =) to 6693,39. And PBM = 5000. Whence PQRM = 1693.39. Therefore by cor. 2. pr. 91. I. force of the spheroid : force of the sphere : : (when P and A coincide) $AS - \frac{AS \times PQRM}{CS^2} : \frac{AS}{3} :: 1 - \frac{PQRM}{CS^2} : \frac{1}{3} :: 1 - \frac{4 \times 1693.39}{10201} : \frac{1}{3} :: 1,008 : 1 ::$ 126 : 125.

[ib. p. 243. if the denſity, &c.] this will appear by conſidering what went before, and cor. 3. pr. 91. I.

[ib. but if the diurnal, &c.] for the (centripetal or) centrifugal force, is as the ſquare of the velocity when the rad. is given.

[ib. 244. — augmented in proportion as the force of gravity is diminiſhed.] for, reviewing the former calculation; if the denſity of the earth were greater than it is; the force of gravity, to the centrifugal force, would be greater than in that proportion of 289 to 1; (and the two diameters would be in a leſs proportion than 289 to 288; and alſo the diameters would be in a leſs proportion than 230 to 229;) conſequently the ratio of the difference of the diameters to either, decreaſes as the gravity increaſes. And on the contrary, that ratio increaſes as the gravity decreaſes. And upon this depends the foregoing proportion, as $\frac{4}{505} : \frac{1}{100} :: \frac{1}{289} : \frac{1}{229}$.

[ib.

[ib. line 11.] read $\frac{29}{5} \times \frac{400}{94\frac{1}{2}} \times \frac{1}{229}$ to 1. In Fig.
the Original, 2d edit. it is $\frac{29}{5} \times \frac{5}{1} \times \frac{1}{229}$ to 1, or as 1 to 8.

[Prop. 20. Whence arifes this Theorem] for let 52.
nkyn be a circle circumfcribed round the ellipfis *nbyl*. Then $kbm : fbc$ ($=$) $abd : : kt^2 : fg^2$. And $ab = \frac{kbm \times fg^2}{kt^2 \times bd}$. But becaufe *kbm* and *kt* are given, and $bd =$ (nearly to) $2kt$ which is given. Therefore $ab \propto fg^2$; but the difference between the weight at *n* and *a*, is as ($at - bt =$) *ab*. Therefore the increafe of weight is as fg^2, or the fquare of the fine of lat. at *a*, nearly, or (by trigonometry) as the verf. of $2an$.

[*ib.* and the arcs of the degrees, &c.] The length of a degree is $\propto$ rad. of curvature, and in the points *n* and *b*, are (by ex. 1. pr. 19. Sect. II. Curve lines) as the parameters of *ny* and *bl*, that is, as bl^3 to ny^3, or 229^3 to 230^3, that is, as 56637 to 57382, as is inferted in the Table, page 247.

But to find the rad. curvature at *b*; we have (by Ex. 2. Prob. 5. Sect. II. Fluxions) $mH = y + \frac{4 \times \overline{a-b}}{abb} y^3$; and therefore (putting *s* for the fine of the angle *ntb*); rad. curvature mC at $b = \frac{y}{s} + \frac{4 \times \overline{a-b}}{abbs} y^3 =$ (becaufe $\frac{y}{s}$ is a given ratio,) $R + \frac{4 \times \overline{a-b}}{abb} Ryy$, very near, R being the radius of curvature in *n*. And therefore the increafe of this radius, which is as the increafe of the degrees, is as *yy* or *ss* nearly. And here $a-b$ being very fmall, $\frac{4 \times \overline{a-b}}{abb} R$, is an extremely fmall quantity.

Fig. [Pr. 23. — which I cannot here descend to explain.] Since the moon's orbit is more excentric than those of Jupiter's satellites, the motion of the moon's apogee will be greater in proportion, than the motion of the apogee in any satellite.

53. [Pr. 27. The area, &c.] For the moment of the area is $\propto cb \times ad$. But $ad \propto$ angle $bcd \times cb =$ horary motion $\times cb$. Therefore, area $\propto cb^2 \times$ horary motion.

[Pr. 28. But the attraction of the moon;] for let A = attraction of P towards T. Let the attraction or force LS be resolved into the two, LM, SM; the first acting towards M the latter towards S. Then the whole attraction of P towards T is $A - ST + LM - TM$. Let $A - ST = F$. Then the attraction of P towards T in the syziges is $F - 2AT$; and in the quadratures $F + CT$. By prop. 25; F : ML : : 178725 : 1000. And the forces 2AT, and CT, will be to each other as 2000 AT, and 1000 CT; that is, as $\frac{2000}{CT \times N}$, and $\frac{1000}{AT \times N}$. And the force F in the syziges and quadratures will be as 178725 directly, and the square of the distance from T reciprocally. Therefore, &c. (Note, $+\frac{2000}{CT \times N}$ is false printed for $-\frac{2000}{CT \times N}$, in line 10, pa. 268.)

[ib. — Quadrature in C; or, which —] for CTP : CTp : : angular motion of the moon from the sun's quadrature : to its angular motion from the fixt point C, that is (because the given circumference, or one revolution, is to be described in either case) : : periodic revolution : synodic revolution.

54. [ib. But by computation I find,] with the radii TA, TC, describe the circles A*lc*, C*ic*. And let AZ

AZ Cz, touch the ellipsis in A and C. And take AI to Ac, and Ci to Cc, as $<$ CTP to CTp (fig. 5. pl. 10.) Now in order to determine the curvature of the orbit, suppose the point *a* to coincide with A, and draw the parts of the orbit ARO, C*ro*, which will pass between the ellipsis and circle; for TO or TE is less than TL, and greater than (TA or) T*c*. And T*o* (that is T*e*) is less than TC or T*c*, and greater than T*l*. Draw the lines TB, T*b*, so that AB = C*b*. And let the points A, I, C, and also C, *i*, *c* coincide; and the curvatures of the ellipsis, orbit, and circles at the points A, C, will be respectively as BE, BR, BI, and *be*, *br*, *bi*. And the difference of curvatures of the ellipsis and circles, and of the orbit and circles in A, C, will be as EI, RI, and *ei*, *ri*, respectively. But RI : EI or OC (because TE = TO, and TI = T*c* : :) $AI^2 : Ac^2$ (Lem. 11. I.) : : $CTP^2 : CTp^2$. Whence $RI = EI \times \frac{CTP^2}{CTp^2} = \overline{BI - BE} \times \frac{CTP^2}{CTp^2}$. Also, *ri* : *oc* or *ei*, or *be* — *bi* : : $Ci^2 : Cc^2$: : $CTP^2 : CTp^2$. Whence $ri = \overline{be - bi} \times \frac{CTP^2}{CTp^2}$. But in the ⊙ TA; BI, AI or AB, and 2AT are $\div$. And in the ⊙ TC; *bi*, C*i* or C*b*, 2CT are $\div$; whence $BI = \frac{AB^2}{2AT}$; and $bi = \frac{Cb^2}{2CT}$. Also in the ellipsis, $BE \times 2AT : BA^2 : : AT^2 : TC^2$. And $be \times 2CT : bC^2 : : CT^2 : AT^2$, Whence $BE = \frac{BA^2 \times AT}{2CT^2}$. And $be = \frac{bC^2 \times CT}{2AT^2}$. Now from hence, the curvature of the orbit at *a* : curvature of the orbit at C : : BR or BI — IR : *br* or *bi* + *ir* : : $\frac{AB^2}{2AT} - \overline{BI - BE}$ Fig. 54.

Fig. $\times \frac{CTP^2}{CTp^2} : \frac{Cb^2}{2CT^2} + \overline{be-bi} \times \frac{CTP^2}{CTp^2} :: \frac{AB^2}{2AT} \times$
$\frac{CTp^2}{CTP^2} - BI + BE : \frac{Cb^2}{2CT^2} \times \frac{CTp^2}{CTP^2} + be - bi ::$
$\frac{AB^2}{2AT} \times \frac{CTp^2}{CTP^2} - \frac{AB^2}{2AT} + \frac{BA^2 \times AT}{2TC^2} : \frac{Cb^2}{2CT^2} \times$
$\frac{CTp^2}{CTP^2} + \frac{bC^2 \times CT}{2AT^2} - \frac{Cb^2}{2CT} :: \frac{CTp^2}{AT \times CTP^2} -$
$\frac{1}{AT} + \frac{AT}{TC^2} : \frac{CTp^2}{CTP^2 \times CT^2} + \frac{CT}{AT^2} - \frac{1}{CT} ::$
$\frac{CTp^2 - CTP^2}{AT \times CTP^2} + \frac{AT}{TC^2} : \frac{CTp^2 - CTP^2}{CT \times CTP^2} + \frac{CT}{AT^2}$
$:: AT^3 + \frac{CTp^2 - CTP^2}{CTP^2} \times AT \times CT^2 : CT^3 +$
$\frac{CTp^2 - CTP^2}{CTP^2} \times CT \times AT^2.$

55. [Pr. 29. The tangent of the angle CTP, &c.] On the axis DC describe the circle C*d*D, and let the line T*b* revolve uniformly to *d*, in the same time that TP revolves to A, and describes areas proportional to the times in the ellipsis CAD. Draw *be* ∥ to *d*T, and draw T*b*, TP. Since the circle and ellipsis are described in equal times, parts proportional to the whole will be described in any equal times; now area ellipsis : area circle : : TA : T*d* (= TC) : : *e*P : *eb* : : (*e*PC + *e*PT =) TPC : (*eb*C + *eb*T =) T*b*C. Therefore the point in the circle is at *b*, when the point in the ellipsis is at P. But TA : (T*d* =) TC : : *e*P : *eb* : : tangent < *e*TP : tangent < *e*T*b* (= < of the mean motion.)

[ib. Square of the sine of the angle CTP.] These things are plain from prop. 26, where P*d* (fig. 4.) is the excess of the moment, and is as $\frac{PK^2}{PT}$.

56. [ib. which we may effect, &c.] Let A be the area described by the moon in any time, arch

CP

CP $= z$, $r =$ radius TC. PK $= y$, TK $= u$, Fig. 56.
$t =$ tan. PTC. Then $\dot{A} = \frac{yy\dot{z}}{r} = y\dot{x}$, and A $=$ Fl : $y\dot{x}$ = area CPK. But TPC expreſſes the mean motion, and therefore TPK is the equation, which is as TK $\times$ PK or uy.

But by the nature of the circle, $\dot{z} = \frac{rr\dot{t}}{rr+tt}$, and ſince $\dot{t}$ is in a given ratio to t, (or as .3123 to 69,) therefore $\dot{z}$ is as $\frac{rrt}{rr+tt}$ or $\frac{t}{\sqrt{rr+tt}} \times$ TK or $y \times$ TK, or as uy. Therefore decreaſing the tangent in the ſubduplicate ratio of 11073 to 10973, or in the ſimple ratio of 69 to 68, 6877; accelerates the area in proportion of PK², as it ought to do.

[ib.—in a proportion compounded of the duplicate, &c.] for (by cor. 16. pr. 66. L. I.) all angular errors are as the ſquare of the time of the moon's revolution, directly, and the ſquare of the time of the earth's revolution inverſly; that is (by pr. 15. L. I.) as the ſquare of the time of the moon's revolution directly, and the cube of the earth's diſtance from the ſun inverſly.

[Pr. 30. And this force by prop. 25, is, &c.] The force 3PK : force ML : : 3IT : PT (for theſe are the ſame) and force ML : centripetal force by which the moon revolves, &c. : : 1 : $178\frac{29}{40}$ (by pr. 25.) therefore *ex equo*, force 3PK : centripetal force the moon revolves with : : 3IT $\times$ 1 : PT $\times$ $178\frac{29}{40}$. Or as IT : rad. $\times$ 59,575.

[ib. the half of which the moon] it ſhould be, which the moon, by the action of the ſaid force, as it is in the firſt edition.

[ib. And the angle PTM is equal to the angle, 57.
&c.] for in this caſe LM is perpendicular to MP. Let PR be a tangent to the point P; then the triangle

Fig. angle RPT is a right one; and angle RPM $=$
57. angle PTM. And angle LPM : $<$ (RPM $=$) PTM (when the radius is PM) : : LM : RM : : force producing LM : force producing RM : : 1 : 59,575.

[Pr. 31. cor. and the decrement is to the remaining motion as 100, &c.] The motion of the nodes in the octants : motion in the syziges : : $11073^2 : 11023^2$: : (because 11073, 11023, 10973 are in arithmetical, and nearly in $\div\!\!\div$ progression) 11073 : 10973; and decrement : remaining motion : : (11073 — 10973 $=$) 100 : 10973.

58. [ib. but the decrement] for the decrements are as the forces and times, or as the whole motions and times; that is, decrement at H (or increment at b) : decrement at A : : mot. at H, and time : mot. at A, and time : : (that is, by what went before) as mot. at H $\times \overline{yy - \frac{1}{2}rr}$: mot. at A $\times \overline{rr - \frac{1}{2}rr}$ (or mot. at A $\times \frac{1}{2}rr$.) This reasoning being very obscure, you will find the motion of the nodes clearly investigated, in (Prop. V. Sect. VI. and cors. of) my Astronomy.

[Prop. 32, it is drawn back again] that is, supposing the place of the node given.

[Prop. 32. Now the area of the semicircle] For let NT $= 1$, AZ $= y$, TZ $= x$. Then $eZ = \frac{y^3}{9.0827 + yy}$, and A$e$ (AZ — eZ) $= \frac{9.0827y}{10.0827 + yy}$.

But $yy = 1 - xx$, therefore $Ae = \frac{9.0827\sqrt{1 - xx}}{10.0827 - xx}$

$= \frac{\overline{b-1}\sqrt{1-xx}}{b - xx} = \overline{b-1} \times \frac{1}{b} + \frac{xx}{bb} + \frac{x^4}{b^3}$

&c. $\times$ into $\sqrt{1 - xx}$, putting $b = 10.08276$, and the fluxion area N$Ae = \overline{b-1} \times \frac{1}{b} + \frac{xx}{bb} + \frac{x^4}{b^3}$, &c.

&c. $\times \dot{x}\sqrt{1 - xx}$; whose fluent (by form 17. Flux- Fig. ions) $= \frac{1}{b} + \frac{1}{4bb} + \frac{1}{8b^3}$ &c. $\times \overline{b-1}$. φ (putting φ = quadrant or $\frac{NAn}{2}$) $= .92435\ \varphi$, therefore half the area NFn = .07565φ. And the area φ (or quadrant) is to the area $\frac{NFn}{2}$; or the semicircle, to the whole area NFn; as 1 to .07565, or as 793 to 60.

[Prop. 33. will nearly agree with] let N = 19 49 3 55, p = periphery of NAn or DFB, E the equation, S = S.2NA.

Then NAZ represents the mean motion of the nodes for the arch NA, as N does for the whole circle; and NAZ is the true motion; and ATZ is the difference proportional to the equation. Divide by $\frac{1}{2}r$, and then $p : \frac{\dot{x}y}{r} :: N : E$, or $p : \frac{2\dot{x}y}{r} :: \frac{1}{2}N : E$, that is, $p : S :: \frac{1}{2}N : E$; whence if BF = 2NA, then (by construction) AD : CD :: $p : \frac{1}{2}N :: S : E$. Suppose S an arch in DF, then 360 : $\frac{1}{2}$N :: degrees in S : E; but deg. in S (in the circle DF) : degrees in DG (in the circle DG) :: AD : CD :: (by construction) 360 : $\frac{1}{2}$N in degrees :: degrees in S : E; whence the degrees in DG = E; or the angle DAG = equation.

[ib. cor.] for in the syziges the areas ANZ and eNZ vanish; and NA is 90°, when they are in the quadratures; and when in the octants, the sine of BF = S.90 = 1; therefore,

38.3	—	1.58319
Rad.	—	10.
1	—	0
S. < A (1° 30′)		8.41681

[Pr.

Fig. [Pr. 2. pa. 291, and by the demonſtration] the angle ATN will be the diſtance from the nodes true place, by conſtruction. Alſo the mean motion of the ſun, in the time NA : mean motion of the node from the ſun, in the ſame time : : (mean annual motion of the ſun : mean annual motion of the node from the ſun : : (by Pr. 1,) area of the ellipſis : area of the circle : :) TBN : TFN; but TBN was the mean motion of the ſun in the time NA; and therefore TFN is the mean motion of the node from the ſun in that time NA; and FTN the angle of the mean motion, or the diſtance of the ſun from the mean plane of the node.

39. [ib. cor.] for let *atn* = *ntb* = 45°, and *fc* ⊥ to *at*; then *fc* is the ſine of *fta*, the Δ's *fcb* and *btb* are ſimilar; therefore *fc* : *fb* : : *bt* : *bb*; and by compoſition, *fb* : *ct*, or *at* : : *bf* : *fc*; and by inverſion *fc* is to *at* or *kt* : : *fb* : *fb* or *by* : : *k*H : H*m* or *tk* + *t*H.

[ib. but the ſine of] draw *oz* ⊥ to it, then *oz* is to *fc* in a ratio compounded of (*or* to *fb* or of) *os* to *fg*, and (the ſine of *orz* or *trs* to the ſine of *fbc* or *tbg*, that is, of) *ts* to *tg*; that is, in the compound ratio of *os* × *ts* to *fg* × *tg*, that is (becauſe rad. : coſine : : 2ce ſine : ſine of the double arch, and therefore the rectangle of the ſine and coſine being as the ſine of the double arch) as ſine of 2*otn*, to ſine of 2*ftn* or radius.

[ib. Sch.] for TS : (TS + SK =) TK : : 360° : (360° + 39°,6355 =) 399,6355 : : 9,0827646 : 10,0827646. Therefore TH : TK : : $\sqrt{9{,}0827646 : 10{,}08}$, &c. And therefore TH : HK : : 18,0324761 : 1 : : TS : SH : : mean motion of the ſun : mean motion of the node = 19° 18′ 1″ 23‴$\frac{2}{3}$.

Laſtly, TK + TH (= 38,22428 : KH (= 1) : : rad. : ſine of 1° : 29′ : 57″ (by cor.)

[Pr. 34. cor. 2. with due regard] for whilſt *p* moves from Q to F, the ſum of the areas is comprehended

prehended by TH produced, FQ, and tangent to Q; but in moving from F to q, the line Hp falls on the other ſide of the circumference, and the ſum of theſe areas is that comprehended by FT produced, Fq, and the tangent to q; and the difference of theſe areas is the ſemicircle, generated in the time of deſcribing QAq; and in the time of deſcribing the whole circumference, the whole circle will be generated. Fig.

[ib. cor. 4. that is, as the diameter] Suppoſe Qp to be the double diſtance of the moon from the quadratures; MK the ſine thereof. Then Kk : Mp : : MK : radius. And the ſum of all the Kk's, or the diameter : ſum of all the Mp's or QAq : : ſum of all the ſines : ſum of as many radii, or half as many diameters. And therefore (multiplying the antecedents by $\frac{Pp}{PG}$, and the conſequents by 2) the ſum of all the ſines $\times \frac{Pp}{PG}$: ſum of as many diameters : : diameter $\times \frac{Pp}{PG}$: whole circumference.

[Prop. 35. by the ſame increments as the ſine of inclination doth, by cor. 3.] For if AEG be double the diſtance of the nodes from the quadratures, GED will be double the diſtance of the nodes from the ſun; and they both have the ſame ſine.

[ib. Schol.] Here are ſeveral particulars about the moon's motion barely laid down; many of them being taken only from obſervations.

[ib. pa. 299. The force of this action is greater] It is greater the nearer it is, and attracts the moon from the earth, and ſo dilates the orbit the more. But the further off, the leſs force, and the leſs the moon is drawn from the earth. And at a greater

diſtance

Fig. diſtance ſhe moves ſlower; and at a leſs diſtance, faſter.

[ib. further I found that the apogee] by cor. 14. prop. 16. B. I. And if a be the ſun's mean diſtance, and $a+x$ any other diſtance, then the motion will be as $\frac{1}{\overline{a+x}^3}$ or $\frac{1}{a^3}-\frac{3x}{a^4}$, or $\frac{1}{a^3}\times\overline{1-\frac{3x}{a}}$, and the equations, as $\frac{x}{a}$, or as x.

The motion of the ſun, that is the angular motion, is reciprocally as the ſquare of the diſtance.

[ib. but if the motion,] let v be the velocity, m the mean motion of the ſun. Then if v be as $\frac{1}{\overline{a+x}^2}$ or $\frac{1}{aa}-\frac{2x}{a^3}=m-\frac{2x}{a}m$; then the equation is $\frac{2x}{a}m$. But if v be as $\frac{1}{\overline{a+x}^3}$ or $\frac{1}{a^3}-\frac{3x}{a^4}=m-\frac{3x}{a}m$, then the equation is $\frac{3x}{a}m$. And the firſt equation to the ſecond, is as $\frac{2x}{a}m$ to $\frac{3x}{a}m$, or as 2 to 3. And hence the greateſt equation of the apogee and nodes (ſum of all the $\frac{3x}{a}m$): their mean motion (m) : : 2° 54′ 30″ : ſun's mean motion; the greateſt equations being as the mean motions.

[ib. pa. 300. By the theory of gravity I likewiſe found] For then the moon being nearer the ſun (in all points of its orbit taken together), the ſun's force muſt be greater.

[ib. p. 301. By the ſame theory of gravity the action,] For when the ſun is in the line of the nodes, its whole force is exerted in moving the moon, and none in moving the plane of its orbit.

[ib.

[ib. By the ſame theory of gravity, the moon's] Fig.
By cor. 8. prop. 66. B. I.

[Pr. 36, in other poſitions of the ſun,] by the reaſoning in cor. 19. pr. 66. L. I. depreſſion of the water at P : aſcent at P in the direction PH : : (LM : TM : :) PT : 3PK; and aſcent at P in direction PH : aſcent at P in direction TP : : (rad. : S.PTK : :) PT : PK; therefore (*ex equo*) depreſſion at P : perpendicular aſcent there : : (PT^2 : $3PK^2$: : $\frac{1}{3}PT^2$: PK^2; but the force PT that depreſſes P is given, therefore the force to raiſe the waters at P, is always as PK^2, that is, as the verſed ſine of 2PTC or twice the ſun's altitude.

Alſo the effects of theſe forces (by cor. 14. pr. 66. L. I.) at different diſtances from the ſun are reciprocally as the cubes of theſe diſtances; and therefore,

Cor. 1. The ſun raiſes the water under it to one Paris foot and 11 $\frac{1}{30}$ inches.

[Pr. 37, but becauſe of the reciprocation] ſee pr. 24.

[ib. and the ſun's force in—] let A be the po- 60.
ſition of the moon, and ATP = angle the ſun and moon makes at the earth's center; and let PT repreſent the force of the ſun, which divide into the forces PK, KT, PK acting in the direction of the moon increaſes her force, KT acting in a different direction, diminiſhes it; and therefore PK — KT (or the difference of the ſine and coſine of the angle ATP) is the abſolute increaſe of the moon's force; but this differs not much from the coſine of double the $\angle$ ATP, (in an angle of $18°\frac{1}{2}$ the difference = ,6310189) and in the arches 0° 45° 90° they perfectly agree. Or thus,

Let *qc* = moon's force, *ca* = ſun's force. If the 61.
ſun is in *n*, let $\angle$ *ace* = 2 $\angle$*acn*, then *qe* or *qb* = ſum

Fig. 62. ſum of the forces, *cb* = ſun's force, in that poſition = coſ. of 2 < *acn*.

[ib. But further, the force of the moon] for let PT be the whole force of the luminary in the place P (or in D) the other ſide of (the earth or) its parallel; the force DT = the two forces DK, KT. When the luminary is in D, the force to move the water in the direction TP, is TK; and the difference of theſe, or (TP — TK =) PK is the difference of the forces in the points P, D; or the abſolute force to move the ſea at the lat. CTP. But PK is ∝ □ ſine of ATP, or as □ coſ. CTP.

Then ſince L + S is the whole force when the moon is near the ſyziges, and L — S in the quadratures. Therefore 0,8570327 L muſt be ſubſtituted for L only in the quantity L — S, which repreſents her force in the quadratures.

[ib. from whence we have] alſo 1,017522 L muſt be ſubſtituted for L, in L + S, which repreſents her force in the ſyziges; and 0,9830427 L for L, in L — S, when ſhe is in the quadratures; for theſe repreſent her attractive forces in the ſyziges and quadratures.

[Pr. 38. as the accelerative gravity] for if the diameters be given, the forces to raiſe the water will be ∝ abſolute forces; and if the abſolute forces are given (the ſpheres will be reduced into ſimilar ſpheroids, for) the perturbating forces will be as the diameters (for the perturbating force is ∝ PT fig. pr. 25.) Therefore if neither be given, the forces to raiſe the water in the moon and earth, are in the compound ratio of the earth to the moon, and the moon's diameter to the earth's diameter; or as 1090 to 100; therefore the water riſes to $93\frac{3}{4}$ feet in the moon.

No notice is taken here, of the tides being leſs by reaſon of the moon's motion round the earth, than if ſhe ſtood ſtill in a given poſition. Since the

the motion of the tides reaches 90° for every ebbing and flowing; it would ſeem, that the ſaid motion could not be propagated thro' 90° to its greateſt height, in the time the moon ſtays in the meridian. But (by a computation from pr. 44. II.) it appears, that water will oſcillate in a canal of 90° in length in $\frac{2}{3}$ of an hour; and therefore in that time the tides will be propagated to that diſtance, and riſe to their higheſt; but the poſition of the moon in that time is not ſenſibly altered; and therefore the tides would riſe no higher, tho' ſhe were always to ſtand in a given poſition. But this ſuppoſes that there is depth enough of ſea, to ſupply water ſufficient for the purpoſe. Fig.

[Lem. 1. to Pr. 39. to recede towards this ſide and that ſide] that is, all the parts of AC to act in a direction towards the ſun; the parts of CE, directly from it.

[Lem. 2. The matter in the circumference of every circle IK] by this is meant the circular ring (in the plane of the circle IK) comprehended between the ſurfaces of the ſphere and ſpheroid.

[Lem. 3. about the ſame axe] he means about its diameter.

[ib.] I. Let ABD be a ſpherical ſurface, and AEFD a circumſcribing cylindric ſurface, and their common thickneſs the infinitely ſmall line B*b* or A*a*. Divide AC into an infinite number of equal parts $= in = \dot{x}$, and erect the planes *id*, *ne*; then the ſurfaces of the ſphere and cylinder comprenended thereby are equal. Now, flux. mot. cyl. ſurface : flux. mot. ſph. ſurface : : (as their matter $\times$ velocity, and the matter is given, and therefore, as) $CB \times \dot{x}$, or $id \times \dot{x} : uf \times \dot{x}$. And mot. cyl. ſurface : mot. ſph. ſurface : : as all the $CB \times \dot{x}$: to all the $uf \times \dot{x}$: : AD^2 : area of the circle ABDG. 63.

Again. Let ABD be a ſphere, and AEFD its circumſcribing cylinder; ECA a cone, all revolv- 64.

Fig. ing round AD. Divide BC into an infinite num-
64. ber of equal parts $= bd = \dot{x}$. Thro' the points draw ſpherical ſurfaces *bgd* concentric to ABD, and cylindric ſurfaces *bf* terminated at the line EC. Then (by what went before) mot. cyl. ſurface *bdf* : mot. ſph. ſurface *bdg* : : AD^2 : circle ABDG. Therefore mot. all cyl. ſurfaces, or the ſolid EBC : mot. all the ſph. ſurfaces, or the ſphere : in the ſame given ratio of AD^2 : to circle ABDG.

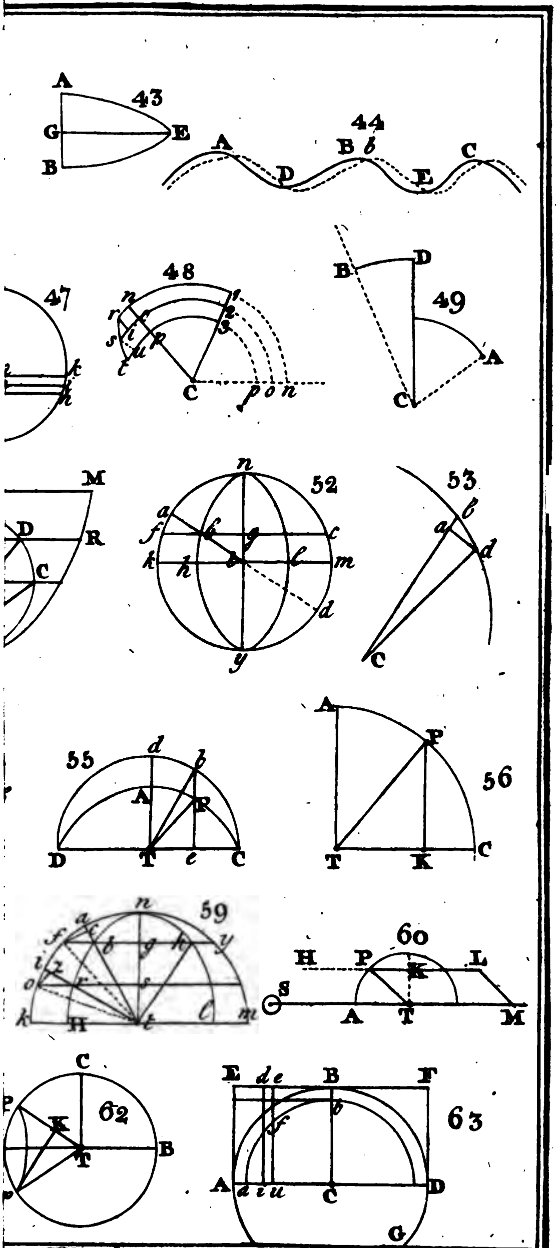
65. Further, flux. mot. cyl. round its axis $\propto$ flux. mot. $\times$ velocity, that is $\propto ef^2 \times \dot{x}$, or $x^2\dot{x}$. And the whole motion is $\propto \frac{x^3}{3}$. And therefore the motion of a cylinder round its axis $\propto$ length $\times$ cube of its diameter, $\times$ denſity; or as its weight $\times$ diameter.

66. Laſtly, let EBCA be a cylinder, and ECA a cone. Then flux. cyl. mot. : flux. con. mot. : : $AE^3 \times \dot{x} : ba^3 \times \dot{x} : : CA^3 \times \dot{x} : Cb^3 \times \dot{x}$, or as $r^3\dot{x} : x^3\dot{x}$. Mot. cyl. round its axis : mot. cone round its axis : : $r^3x : \frac{x^4}{4}$: : (when $x = r = CA$) 4 : 1. Therefore, mot. cyl. : mot. fig. EBC ($=$ difference of mot. of cylinder and cone) : : 4 : ($4 - 1 =$) 3. hence it follows,

Mot. cyl. : mot. ſol. EBC : : 4 : 3
Mot. ſol. EBC : mot. ſphere : : diameter □ : circle,
Therefore, *ex equo*,
Mot. cylinder : mot. of ſphere : : 4 diam. □ : 3 circle.
Or,
Mot. ſphere : mot. cylinder : : 3 circles : 4 diam. ſquare.

67. II. Flux. mot. cyl. : flux. mot. of a ring (or periphery) of the ſame diameter and matter : : (as velocity $\times$ flux. matter, as) $x^2\dot{x} : rx\dot{x}$. And mot. cyl. : mot. ring : : $\frac{x^3}{3} : \frac{rx^2}{2}$: : (when $x = r$) 2 : 3.

And



Pl: IV. pa: 98.

And mot. cyl. : mot. any ring : : 2 cylinders : 3 rings. Fig.

III. Let ED $= z$; and let $\dot{z}$ be given; then 68.
$y\dot{z} = r\dot{x}$. Now flux. mot. ring round C : flux. mot. ring round AB : : $r\dot{z}$: $y\dot{z}$ or $r\dot{x}$. Therefore mot. ring round C : mot. ring round AB : rz ; rx : : (when $x = r$) AD : AC : : circumference : 2 diameters.

Therefore from the 1, 2, 3 Art it follows, *ex equo*, that the motion of a ſphere : motion of a ring round their axes : : as the matter in the cylinder and ring, and 6 × circumference × circle : 24 diam. cube : : or as ſquare of the quadrantal arch : to ſquare of the diameter : : that is, as matter in the cylinder to matter in the ring, and 61685 : to 100000; or as the matter in the ſphere to the matter in the ring, and 925275, to 1000000.

[Pr. 39. (for the reaſons above explained),] for (by cor. 2. pr. 30.) mean horary motion of the nodes : $16'' \; 35''' \; 16'''' \; 36^{v}$: : AZ^2 : AT^2. And the ſum of all the mean horary motions throughout the year : ſum of as many $16'' \; 35'''$, &c. : : ſum of all the AZ^2 : ſum of as many AT^2 : : (by Lem. 1. pr. 39.) 1 : 2; for the diurnal rotation, and annual motion of the earth, are ſuppoſed uniform.

[ib. But becauſe the plane of the equator.] For by prop. 30, the diſturbing force is as 3PK, (and in that prop. the inclination of the planes being ſmall is not taken notice of,) but when the plane of the orbit of P is inclined to the ecliptic, the line PK is projected into pK, and the force will therefore be diminiſhed, in the ratio of PK to pK, or as the radius to the coſine of inclination; for the difference of the diſtances of P and T from the ſun become leſs in that proportion, and the difference of the forces are diminiſhed in the ſame ratio.

 This

Fig. This being a propofition of great difficulty, depending upon very intricate calculations, which few people are judges of; for this reafon, the Author's demonftrations have been objected to, and cenfured by fome people as not true. And Mr. Simpfon (in his Mifcellaneous Tracts) mentioning fome of thefe Cavillers, falls into the fame notion. He has invented a fort of motion (which he calls *Momentum*) unknown to Sir I. Newton, or any body elfe, and which differs from Sir Ifaac's, in the ratio of 800000 to 925725. But I never before heard of any motion that was not made up of the quantity of matter and velocity.

Another frivolous objection he makes, is about the motion of a ring being different from that of the equator. And he tells us, that the motion (*Momentum*) of a ring round its diameter is only half of what it would be, when revolving in its plane round the center. But it is more than half, for it is as 1 to $\frac{3.1416}{2}$, as is demonftrated in Art. III. What he writes afterwards (tho' he fays, *it is evident*,) is not intelligible. But he concludes at laft in his way, that Sir I. Newton has made the preceffion (by the fun's force) to be but half of what it fhould be. And as fome he mentions had made the whole propofition erroneous, he modeftly afcribes but *two miftakes* to Sir Ifaac in this one propofition. But I believe, that whoever reads the foregoing Notes, will foon be convinced, that his demonftrations are all right, and that all thefe blunders they tell us of, are entirely of their own making, and muft be afcribed to themfelves only.

[Lem. 4. p. 326. Wherefore if both the quantity of light,] for quantity of light is as the quantity the body receives from the fun directly, and the fquare of the diftance from the body reciprocally;

cally; that is, $\propto$ square of the apparent diameter directly, and square of the body's distance from us, reciprocally: therefore the body's distance is as the apparent diameter directly, and $\sqrt{\text{light}}$ reciprocally. Fig.

[Pr. 40: cor. 4.] For then the parabola and earth's orbit touch in the vertex, and the area there will be $= \frac{1}{2}$ velocity $\times$ distance or radius or $\frac{1}{4}$ parameter, that is $= \frac{10000 \times 243.2747}{2}$, and $\frac{10000 \times 10.1364\frac{1}{2}}{2}$ for the diurnal and horary motion. The rest is plain from Pr. 14. L. I.

[Lem. 5.] For the ordinates AH, BI, CK, &c. Put a, $2a$, $3a$, &c. and take the differences as follows. 69.

$$\begin{array}{ccccccccccc}
a & & 2a & & 3a & & 4a & & 5a & & 6a \\
& b & & 2b & & 3b & & 4b & & 5b & \\
& & c & & 2c & & 3c & & 4c & & \\
& & & d & & 2d & & 3d & & & \\
& & & & e & & 2e & & & & \\
& & & & & f. & & & & &
\end{array}$$

Now, to find any ordinate as $4a$, we have $4a =$

$$\begin{array}{lll}
3a - 3b = 2a - 2b & = & a - b \\
\quad\quad\quad\quad - 2b + 2c & & -2 \times b + 2 \times c \\
& & \quad\quad\quad + c
\end{array}$$

$+ d = a - 3.b + 3.c - d.$
and so of others.

Fig. 69. Case I. If we substitute for p, q, r, &c. their equals, we shall have

$$\begin{aligned} RS = {} & a \\ & + b \times -SH \\ & + c \times -SH \times -\frac{IS}{2} \\ & + d \times -SH \times -\frac{SI}{2} \times \frac{SK}{3} \\ & + e \times -SH \times -\frac{SI}{2} \times \frac{SK}{3} \times \frac{SL}{4} \\ & + f \times -SH \times -\frac{SI}{2} \times \frac{SK}{3} \times \frac{SL}{4} \times \frac{SM}{5} \text{ \&c.} \end{aligned}$$

Now let RS fall upon any ordinate, as for example on LD, then SH, SI, SK, SL, &c. will be $=$ LH, LI, LK, o, &c. $= 3, 2, 1, o$, &c. respectively, which being written in the aforesaid series, we have

$$RS = a - 3.b - 3 \times \frac{-2}{2} c - 3 \times \frac{-2}{2} \times \frac{-1}{3} d$$

$+ o e = a - 3.b + 3.c - d$, which, by what went before, is the value of the ordinate DL; and since this holds generally, it is plain, this series will give the value of RS wherever it falls.

Case II. Let x represent any base HS, y any ordinate RS. Assume $y = A + Bx + Cx \times \overline{x - P} + Dx \times \overline{x - P} \times \overline{x - Q}$, continued to as many terms as there are ordinates, which suppose four; P, Q, R, &c. being respectively $=$ HI, HK, HL. Then putting AH, BI, CK, DL, successively for y, and o, HI, HK, HL for x, we shall have from the general equation, $AH = A$, $BI = A + BP$, $CK = A + BQ + CQ \times \overline{Q - P}$, $DL = A + BR + CR \times \overline{R - P} + DR \times \overline{R - P} \times \overline{R - Q}$, that is, from the figure,

AH

$AH = A = a$, Fig. 69.

$BI = A + B \times HI$.

$CK = A + B \times HK + C \times HK \times IK$.

$DL = A + B \times HL + C \times HL \times IL + D \times HL \times IL \times KL$.

Whence by Subſtraction.

$AH - BI = -B \times HI$,

$BI - CK = -B \times IK - C \times HK \times IK$.

$CK - DL = -B \times KL - C \times \overline{HK + IL} \times KL - D \times HL \times IL \times KL$.

Then dividing by the coefficients of B.

$\frac{AH - BI}{HI} = -B = b$.

$\frac{BI - CK}{IK} = -B - C \times HK = 2b$.

$\frac{CK - DL}{KL} = -B - C \times \overline{HK + IL} - D \times HL \times IL = 3b$.

Again by Subtraction.

$b - 2b = C \times HK$.

$2b - 3b = C \times IL + D \times HL \times IL$. And dividing.

$\frac{b - 2b}{HK} = C = c$.

$\frac{2b - 3b}{IL} = C + D \times HL = 2c$. Alſo

$c - 2c = -D \times HL$, and

$\frac{c - 2c}{HL} = -D = d$.

Thus the coefficients A, B, C, &c. are determined for four ordinates, and they are found the ſame way, if more ordinates are given. Then taking x at pleaſure, as ſuppoſe $=$ HS, then RS or y will be $a + bx + cx \times \overline{x - P} + dx \times \overline{x - P}$

Fig. $\times \overline{x - Q}$, &c. $= a + b \times p + c \times p \times IS + d \times p \times IS \times SK$, &c. $= a + bp + cq + dr$, &c.

69\. [Cor. Lem. 8.] for then AC : Aδ ($=$ AI $-$ δI) : : AμC :] A$\delta dy - d\mu x$ ($=$ AIμy—δIμX), and : : ASC : ASδ (Propor. 10.) : : (AμC $+$ ASC $=$) ASCμA : (Aδdy $- d\mu x +$ ASδ $=$ ASδdyA $- d\mu x$ $=$ (becaufe Δ's $x\delta\mu$ and δSμ are equal) ASδdyA $- d\mu x + x\delta\mu$ $-$ S$\delta\mu$ $=$ AS$\delta\mu dy$A $-$ S$\delta\mu$ $=$) ASμdyA : : time of defcribing AμC : time of defcribing A$y\mu$.

[ib. Schol.] For Bn will cut the chord AC in a point a little beyond E (towards C), fuppofe at e. Now the area AEXμA to the area ACYμA, is in fomething a greater ratio than AE to AC, as fuppofe as Ae to AC, more near than before. Therefore ASEXμA : ASCYμA : : Ae : AC. Whence ASBXμA (or ASEXμA) : ASCYμA : : Ae : AC. Or ASBYμA : ASCYμA : : (Ae : AC : :) time in AB : time in AC, very near.

[Lem. 10. to the triangle ASC, that is,] ACμA : ASC : : AC $\times \frac{2}{3}$Mμ : AC $\times \frac{1}{2}$SM (becaufe the $<$'s SMA and AIμ are equal, and Mμ $=$ Iμ.) : : AC $\times$ $1\frac{1}{3}$Mμ : AC $\times$ SM : : MN : SM. And by compofition ASCμA : ASC : : SN : SM.

[Lem. 11. Subduplicate proportion of 1 to 2,] and therefore $= \frac{AC}{\sqrt{2}}$. And the arch in half the time $= \frac{AI}{\sqrt{2}}$. And $\frac{AI^2}{2} \div 2SP = \frac{AI^2}{4SP}$ $=$ fpace defcribed in defcending.

[Prop 41. and BE, by Lem. 11, is a portion,] tS and this hypothenufe (mentioned in the prop.) are the diftances of the earth and comet from the fun. Now BE to tV is compounded of BE to the part of the hypothenufe (projected into BE) or of BS to the whole hypothenufe, and of that part of the hypothenufe to tV, that is, as the gravitating

forces

forces at the comet and earth (for theſe lines would be deſcribed by falling bodies in equal times, by theſe forces), that is, as St^2 to hypothenuſe ſquare. And therefore BE : tV : : BS $\times$ St^2 : hypothenuſe cube. Fig.

Further, becauſe of the immenſe diſtance of S, any of the points B, i, μ, are nearly in the curve of the parabola, and μ the vertex, and μp, $i\lambda$, $q\sigma$ nearly parallel. Therefore $\angle I\mu i$ = (by reaſon of the rectangle $Ii\mu\lambda$)] $\angle p\mu L$ or = $S\mu L$. Therefore μI is the diameter to the vertex μ; alſo $\xi\sigma$ = (by conſtruction) $3S\sigma + 3i\lambda = 3S\sigma + 3I\mu = 3S\sigma + 3\mu p$ = (becauſe $2O\sigma = 3\mu p$) $3S\sigma + 2O\sigma$. And therefore $\xi O = 3S\sigma + 3O\sigma = 3OS$. But μO : $I\mu$: : $p\tau$: Ip. But μI is biſected by $i\lambda$; therefore $Ip = 2I\lambda$; and $p\sigma = I\lambda = \frac{1}{2}Ip$ (ſince $I\sigma = 3I\lambda$ by conſtruction); therefore $\mu O = \frac{1}{2}\mu I$. Whence (by Lem. 8.) a line drawn from ξ divides the chord AC nearly as the time; whence BE drawn towards ξ is rightly drawn. But (by conſtruction) new BE : former BE : : BS^2 : $\overline{S\mu + \frac{1}{3}i\lambda}|^2$: : as the gravitating force at the diſtance $S\mu + \frac{1}{3}i\lambda$: to gravitating force at the diſtance B or μ : : ſpace fallen thro' at the diſtance $S\mu + \frac{1}{3}i\lambda$, in the time of deſcribing the arch $\frac{1}{2}A\mu C$: ſpace fallen thro' at B or μ, in the ſame time. But former BE = ſpace fallen thro' at μ. Therefore new BE = ſpace fallen thro' at the diſtance $S\mu + \frac{1}{3}i\lambda$, (in half the time the comet deſcribes the arch intercepted between TA, τC = (by Lem. 11.) $i\lambda$, or this new BE nearly; which therefore is rightly determined, nearer than before. 70

Becauſe the diſtance of the ſun and comet is ſomething more than the hypothenuſe of the triangle, whoſe baſe is $S\mu$ and perpendicular IO (becauſe O is only a point in the chord, which ſhould be in the arch), therefore he ſuppoſes that diſtance = hypothenuſe (whoſe baſe is $S\mu + \frac{2}{3}i\lambda$, and perpendicular

Fig. 70. pendicular IO, that is) $=$ DO. Alſo MP : X : : (by conſtruction) $\sqrt{St} : \sqrt{OD}$: : (by cor. 6. pr. 16. I.) velocity of the comet at diſtance DO : velocity thereof at diſtance St : : ſpace deſcribed (uniformly) at the diſtance DO, and in the time of deſcribing the arch ABC : ſpace deſcribed (uniformly) at the diſtance St, in the ſame time, which is $=$ X by conſtruction; and therefore MP $=$ ſpace deſcribed (uniformly) at the diſtance DO, (in the time that the arch ABC is deſcribed) $=$ (by Lem. 10. cor.) to the chord of that arch.

The points e, a, c, g; and ε, α, χ, γ, being found out as E, A, C, G, before; is with an intent to find at laſt MP $=$ MN, or AC $=$ AG. Wherefore ſeveral of them being thus found, and a ſimple curve (viz. a circle) drawn thro' theſe points, finds the points Z and X where thoſe lines would be equal; for the nearer they approximate to equality, the nearer they come to the true points of the comet's orbit.

Further, velocity of a comet at Q (in a parabola) : velocity of a comet at Q (in a circle) by cor. 7. pr. 16. I.) : : $\sqrt{2} : 1$: : (nearly as) $\frac{4}{3} : 1$. Alſo velocity at Q (in a circle) : velocity at t (in a circle, by cor. 6. pr. 4. I) : : $\sqrt{St} : \sqrt{SQ}$. Therefore, *ex equo*, velocity of the comet at Q, in a parabola : velocity of the earth in its orbit at t : : $\frac{4}{3}\sqrt{St} : \sqrt{SQ}$. But velocity of the comet at Q : velocity of the earth at t : : (nearly as) AC : $T\tau$. Therefore AC : $T\tau$: : $\frac{4}{3}\sqrt{St} : \sqrt{SQ}$; or AC : $\frac{4}{3}T\tau$: : $\sqrt{St} : \sqrt{SQ}$. As it is by conſtruction (for AC is to $\frac{4}{3}T\tau$ in the reciprocal ſubduplicate ratio of SQ to St, and not in the direct ſubduplicate ratio, as is falſely printed; therefore Q is (nearly) in the chord of the parabola, and B a point of the comet's orbit, nearly.

Laſtly, if MP $=$ MN, or AG $=$ AC. Then Yb : YB : : Yc : YE : : ac : AC : : velocity in b : velo-

velocity in B : : $\surd SB : \surd Sb$. But if $Sb = SB$ Fig.
and MP or AG invariable; it will be $Yb : YB$: : as 70.
ac or $AG : AC$, when the point G falls in CY. Therefore universally $Yb : YB : : AG \times \surd SB : AC \times \surd Sb : : MP \times \surd SB : MN \times \surd Sb$, to find the point b true.

[ib. p. 352. may be seen in the following table.] The places of the comet in this and all the other tables, are the geocentric longitudes and latitudes of the comet.

[ib. p. 358. And thinking, &c.] In plate 18, while the comet passes thro' I, K, L, M, &c. the earth passes from between P and K towards G. Whence the comet will move swift thro' LKL, and NOP; slower thro' LBM, and QRS, as is said in the foregoing page 357.

[ib. p. 360, as about 6 to 1000,] for by page 348,9. $\frac{\text{lat. rect.}}{4} = 6$, nearly; when earth's dist. $= 1000$.

[ib. p. 366. I found, that at the height, &c.] 71.
Let SA = rad. earth = 20950000 English feet, nearly, AB = 850, AF = AS, and the densities AH and BI are as 33 to 32. But (by cor. 2. p. 22. II.) $Aa - Bb : Aa - Ff : : thin : thnz$; that is, $AB \times SF$, or $2AB \times SA : AF \times SB$, or $SB \times SA$: : $thin : thnz$; or $2AB : SB : : thin : thnz$; that is, 1 : 12324 : : $thin : thnz$: : (by schol. prop. 86. hyperbola,) log. St — log. Su : log. St — log. Sz : : ,013364 : 1.518514 — log. Sz. Whence log. Sz = — 164.820578; and SZ or FN = , ($0^{163 \text{ cyphers}}$) 66 +. Wherefore FN : AH : : ,0^{163} 66 : 33 : : or as 2 : 10^{165} : : 1 : 10^{164} : : density in F : density in A. But diameter of Saturn's orb is = 191 times the semidiameter of the orb's magnus = 191×20000 radii of the earth = $191 \times 20000 \times 20950000 \times 12$ inches = 96 0^{13} + inches. And a sphere of 1 inch diameter : to this sphere : : as 1 :

Fig. $\overline{960^{13}}|^{\frac{1}{2}} : : 1 : 88\,0^{43} +$; but $10^{164} > 8\,0^{44}$. Therefore, &c.

[Prop. 42.] the correction of the orbit here given is computed by the rule of falſe; 1ſt, for the longitude of the node, by operation 1 and 2, and 2dly for the inclination of the orbit, by operation 1 and 3.

I. for the longitude of the node; ſince (by conſtruction) time of deſcribing D : time of deſcribing E : : D : E : : G : 1; and (time of deſcribing E, nearly $=$) time B : time A : : 1 : C; therefore (*ex equo*) time of deſcribing D : time A : : G : C; therefore G — C is the error of the times between the firſt and ſecond obſervations, for the given longitude of the node K, and the given inclination of the orbit I. Again,

Time of deſcribing d : time of deſcribing e : : d : e : : g : 1; and time of deſcribing e (nearly $=$) time B : A : : 1 : C; therefore time of deſcribing d : time A : : g : C; and g — C is the error of time, between the two firſt obſervations, for the longitude K + P, and inclination I; therefore by the rule of falſe, by two poſitions (which ſays, diff. er. : diff. poſitions : : either er. : correction of its poſition), $G - g : G - C : : P : \frac{G - C}{G - g} P =$ the correction of K, or the computation may be thus, ſince to the ſuppoſition K and K + P, the errors between the firſt and third obſervations, in theſe two caſes, are T — S, and t — S; therefore $T - t : T - S : : P : \frac{T - S}{T - t} P =$ the ſame correction of K; therefore mP (is to be) $= \frac{T - S}{T - t} P$ $= \frac{G - C}{G - g} P$; and therefore $mT - mt = T - S$; and $mG - mg = G - C$.

II. For

II. For the inclination of the orbit, it is plain Fig. $T - S =$ firſt error of the times between the firſt and third obſervations, to the given node K, and inclination I; and $r - S =$ error between the firſt and third obſervation, to the node K, and inclination $I + Q$. Therefore (by the rule of falſe), $T - \tau : T - S :: Q : \frac{T - S}{T - \tau} Q =$ correction of I; again this time of deſcribing δ : time of deſcribing $\varepsilon :: \delta : \varepsilon :: \gamma : 1$; and (time of deſcribing $\varepsilon =$ nearly) time $B : A :: 1 : C$: *ergo* time of deſcribing $\delta : A :: \gamma : C$; therefore $\gamma - C =$ error of the times between the firſt and ſecond obſervations, to the node K and inclination $I + Q$; and $G - C$ was the error between the firſt and ſecond obſervations, to the node K and inclination I; therefore (as before) $G - \gamma : G - C :: Q : \frac{G - C}{G - \gamma} Q =$ correction of I; therefore nQ (muſt be) $= \frac{T - S}{T - \tau} Q = \frac{G - C}{G - \gamma} Q$; and $nT - n\tau = T - S$, and $nG - n\gamma = G - C$. Laſtly, from I and II.) $2T - 2S = mT - mt + nT - n\tau$; and $2G - 2C = mG - mg + nG - n\gamma$; therefore the numbers m, n are rightly found.

[ib. and laſtly, if in—] becauſe the longitudes in the firſt and ſecond orbits are K and $K + mP$, therefore the latus rectums will be R and $R + \overline{r - R} \times m$; and becauſe in the firſt and third orbits or planes thereof, the inclinations are I and $I + nQ$; therefore the latus rectums will be R and $R + \overline{\varrho - R} \times n$; or $R + \overline{r - R} \times m$, and $R + \overline{r - R} \times m + \overline{\varrho - R} \times n$.

Or thus; if the longitude K be increaſed (in op. 1, 2.) by P, then (becauſe the diſtance of the peri-

perihelion from the ſun is increaſed accordingly), the latus rectum will be increaſed by $r - R$, and if the longitude be increaſed by mP, the latus rectum will be increaſed by $m\overline{r - R}$. And for the ſame reaſon (in op. 1, 3.) if the inclination be increaſed to nQ, the latus rectum will be increaſed to $N\overline{\varrho - R}$ (for at the increaſe Q, the increaſe of the latus rectum is $= \varrho - R$, ſince the whole is ϱ, or $R + \overline{\varrho - R}$); therefore if both longitude and inclination be increaſed, the latus rectum will be increaſed in both theſe reſpects, and will become $R + mr - mR + n\varrho - nR$.

Alſo when K becomes $K + P$, the tranſverſe becomes $\frac{1}{L + \overline{l - L}}$ or $\frac{1}{l}$; alſo when K becomes $K + mP$ the tranſverſe becomes $\frac{1}{L + m.\overline{l - L}}$, for the ſame reaſon when I is increaſed to $I + nQ$, the tranſverſe is $\frac{1}{L + n.\overline{\lambda - L}}$. Therefore when both longitude and inclination are increaſed, the tranſverſe will then be $\frac{1}{L + ml - mL + n\lambda - nL}$.

[ib. Sch. pa. 385, the comet] for by pa. 360, comet's diſt. : earth diſt. from ☉ :: 6 : 1000 :: diſt. comet in ☉ diameters : (earth's diſt. in ☉ diam. =) 109 ☉ diam. therefore comet's diſt. from the ſun's center $= \frac{654}{1000} = 654$ diams. and from its ſurface $=,154$ diams. $= \frac{1}{6}$ ſun's diameter.

A DE-

A DEFENCE OF Sir I. NEWTON, AGAINST THE OBJECTIONS that have been made againſt ſeveral Parts of the PRINCIPIA.

Raro antecedentem ſceleſtum,
Deſeruit pede pœna claudo. HOR. Lib. III.

THIS incomparable Treatiſe being written in a conciſe ſtile, and in the ſynthetic method, and being upon ſubjects quite new and untouched before; the generality of readers could make little of it. As it contained a new ſyſtem of Philoſophy, built upon the moſt ſublime Geometry, the greateſt mathematicians were obliged to ſtudy it with great care and attention; and few became maſters of the ſubject; ſo that for a long time it was little read. But at laſt, when the value of it became more known, it gained univerſal approbation; and the whole world ſtood amazed at the numberleſs new diſcoveries contained therein. And upon account of its univerſal agreement with all the phænomena of nature; it was adopted as the only true ſyſtem by all, except ſome few that, thro' envy or ignorance, were bigotted to ſome other ſcheme.

It is true, many things therein depend upon very difficult mathematical computations, not eaſily comprehended by every reader; and this has given occaſion to ſome people to queſtion or even deny the truth of ſeveral propoſitions laid down in this book;

book; and have therefore given computations of their own, different from his, which they assert for truth; and thereby, as they think, proving his to be false. I shall therefore spend a little time in answering the most material of these objections, and shall shew that these principles objected against, are all true in the sense Sir Isaac meant them; and that these objections are only the effect of their own ignorance or inattention.

J. Bernoulli (Tom. IV. pa. 340) thinks Newton's demonstration, of cor. 4th to the laws, is imperfect, where he says, *only the action of two bodies is considered.* And lays down a method, by which he thinks it may be fully demonstrated. But at last leaves it undemonstrated himself; and tells you, that this is the foundation, but leaves the calculation for others to make out.

But Sir I. Newton, after he had shewn the truth of the cor. in two bodies, shews that the common center of gravity of these two bodies and a third, will either be at rest, or move uniformly in a right line. And from thence, that the center of gravity of *any two* (in a system of bodies) suffers no change of state by their actions upon one another. From whence it follows, that in the whole system, all these actions (composed of the action of *every two*) can induce no change. And if the actions of *any two* makes no difference in the center of gravity of the whole, separately considered; then the actions of *every two* considered collectively can make no difference. This is certainly so, tho' delivered in a few words, Sir Isaac was not writing to children.

The same *Bernoulli* (pa. 347) objects against the 10th prop. B. II. for not being solved in a way general enough to please him. Here Sir Isaac has solved the problem according to that law of resistance which really exists; but could not spare

time

time, or fill up his book with uſeleſs enquiries, or things of no conſequence. His noble ſtructure the *Principia* is all gold, but he has left the droſs for ſuch authors as this, who are fond of any thing, and will ſerve well enough for building up their ſyſtems. This author in particular is always depreciating Sir Iſaac, and extolling his own performances; tho' they are long, tedious and laborious to the laſt degree, and often falſe; altho' he has the advantage of the analitical and fluxional methods; and ſeems to be ignorant that the method of analiſis, by which he (*Bernoulli*) ſolves theſe prolems, is incomparably ſhorter than the method of compoſition, in which Sir Iſaac has writ. Had he written the Principia in the analitic way, or according to the method of Fluxions, a ſcience then utterly unknown to the world; it could not have been read by any man living. By the ſame way of reaſoning this boaſting mathematical bully may as well condemn *Euclid*'s Elements, or *Apollonius*' Conics; becauſe moſt of the propoſitions may be demonſtrated incomparably ſhorter by common Algebra.

This Author ſeems to have a particular ſpite againſt the Engliſh mathematicians, being always carping and criticiſing. His works are full of invectives againſt Sir I. Newton for his great diſcoveries, which this low Critic is unſucceſsfully endeavouring to imitate. When one perſon has another man's works to look thro', he muſt be a great blunderbuſs that cannot make ſome ſmall additions; a man placed upon another's ſhoulders, will ſee further than his ſupporter. Yet in many caſes he has diſimproved Sir I. Newton. Sir Iſaac had a work entirely new to execute, in a ſhort, and general way; and could not take notice of every bauble. This author has had nothing to do but to imitate. The regular works of the one will live

thro' all ages; but the confused chaos of the other will sink into everlasting oblivion.

The same author (pa. 484) also objects against the demonstration of the 36th prop. B. II. For says he, *since in Newton's cataract there is no compression of the water in any place, nor against the sides of the cataract, the external water pressing inwards, must disturb the cataract, and mix with it. And therefore the Newtonian explication being contrary to the laws of Hydrostatics cannot subsist.* Yet what Newton says (Case I.) might have satisfied him. His words are; *For let the ice in the vessel dissolve into water, yet will the efflux of the water, as to its velocity, remain the same as before. It will not be less, because the ice, now dissolved, will endeavour to descend. It will not be greater, because the ice now become water, cannot descend without hindering the descent of other water, equal to its own descent.* But this author is eternally cavilling at every thing; and all his whole section on this subject, consisting of about 100 pages, where he has tost this prob. about into all forms, is nothing but a heap of absurd inconsistent stuff, neither agreeable to theory nor experiment. So we shall leave him here drowned in a *gurges* of his own contriving.

In the demonstration of prop. 47, B. II. Sir I. Newton assumes for a principle, that the parts of the air hath a motion excited in it, by some cause or other, according to the laws of an oscillating pendulum. But *Euler* has found fault with him for making such an hypothesis; and has himself assumed some different hypotheses, from which he has pretended to demonstrate, that the motion of the particles of air, are moved according to such hypotheses; and that his demonstration for his hypothesis, is as valid as Newton's is for his. But Sir I. Newton's hypothesis is more than an assumed one, for it is true in fact. For any tremulous or vibrating

of a *fuga vacui* was aſſumed by theſe philoſophers, as the genuine cauſe of theſe effects. So that, upon no better a footing than this, tho' the true cauſe of theſe effects (the external preſſure of the air) be now perfectly known, yet with theſe bigotted people, the ſame falſe principle ſtill exiſts.

Theſe philoſophers (if they can be called ſuch) are endeavouring, contrary to the nature of things, to find out the moſt complicate cauſes for explaining the phœnomena, inſtead of ſeeking the ſimpleſt cauſes; and they ſeem utterly to reject all ſimple cauſes, which are the greateſt beauty of nature. For if their brains were not turned round in a vortex, they could never prefer theſe complex vortical ſchemes before the ſimple doctrine of projectile and centripetal forces. Such Philoſophers!

Sir I. Newton had ſhewn in the ſchol. of Prop. 14. B. III. that the aphelions of the interior planets move a little *in conſequentia*, (by the actions of Jupiter and Saturn) in the ſeſquiplicate ratio of their diſtances from the ſun. But *Bernoulli* ſays this is falſe, for it holds not true in Saturn. And yet Sir Iſaac tells him expreſsly, it is the inferior planets that obſerve that law. So little do ſome men care what they write, if they can only put on an air of contradiction.

In finding the proportion of the axis and diameter of the earth, Prop. 19. B. III. Sir Iſaac aſſumes them to be as 100 to 101, and from thence finds the exceſs of weight, in one above the other, to be $\frac{4}{505}$ parts. Therefore he makes this proportion $\frac{4}{505} : \frac{1}{100} :: $ the force $\frac{1}{289} : \frac{1}{229}$ the true exceſs of height. Yet ſome people have objected againſt this as a wrong way. But herein they are much miſtaken; for if any force be increaſed, the effect will be increaſed in the ſame ratio; and there-

account of them, and ſo ſave the labour of conſtructing theſe uſeleſs vortices.

Another author that is *vortex-mad*, is *Euler*, and he ſeems to go beyond any of the reſt, for he cannot account for the riſing of the tides without vortexes. But he has not ſhewn us by what extraordinary mechaniſm, or inviſible wheelwork his vortices are conſtituted, ſo as to be able to produce the tides, or cauſe the motion of the comets, ſince theſe vortices muſt all run counter to one another, and penetrate one another, and yet miraculouſly preſerve their motions entire.

This idle notion was firſt introduced in the time of ignorance, upon the ſuppoſition of a *fuga vacui*, or that nature abhorred a *vacuum*. But by our better acquaintance with the nature and properties of body, and the laws of motion; we now know, that the operations of nature cannot be performed in a *plenum*; and therefore a *vacuum* is abſolutely neceſſary.

If we had known of no celeſtial bodies but the planets moving all one way, the ſuppoſition of a vortex to carry them about, clumſey as it is, might have paſſed for poſſible. But one would have thought, that the comet's moving all manner of ways, would have cured this delirious notion; and have taught them the impoſſibility of ſuch a ſcheme. But theſe authors, theſe defenders of vortices, are ſo hardy, that they are not at all afraid of an abſurdity or a contradiction. Theſe things do not affect or touch them in the leaſt; but they go on unconcerned in their uſual way, tho' contrary to all the laws of nature.

Did I ſay that the theory of vortices had its original from the principle of a *fuga vacui?* I did. And the principle of a *fuga vacui* had its riſe from the phœnomena of water riſing in pumps and ſyphons, as in the *Torricellian experiment*. Here for want of trying ſuch experiments to their full extent, the principle

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of a *fuga vacui* was assumed by these philosophers, as the genuine cause of these effects. So that, upon no better a footing than this, tho' the true cause of these effects (the external pressure of the air) be now perfectly known, yet with these bigotted people, the same false principle still exists.

These philosophers (if they can be called such) are endeavouring, contrary to the nature of things, to find out the most complicate causes for explaining the phoenomena, instead of seeking the simplest causes; and they seem utterly to reject all simple causes, which are the greatest beauty of nature. For if their brains were not turned round in a vortex, they could never prefer these complex vortical schemes before the simple doctrine of projectile and centripetal forces. Such Philosophers!

Sir I. Newton had shewn in the schol. of Prop. 14. B. III. that the aphelions of the interior planets move a little *in consequentia*, (by the actions of Jupiter and Saturn) in the sesquiplicate ratio of their distances from the sun. But *Bernoulli* says this is false, for it holds not true in Saturn. And yet Sir Isaac tells him expresly, it is the inferior planets that observe that law. So little do some men care what they write, if they can only put on an air of contradiction.

In finding the proportion of the axis and diameter of the earth, Prop. 19. B. III. Sir Isaac assumes them to be as 100 to 101, and from thence finds the excess of weight, in one above the other, to be $\frac{4}{505}$ parts. Therefore he makes this proportion $\frac{4}{505} : \frac{1}{100} :: $ the force $\frac{1}{289} : \frac{1}{229}$ the true excess of height. Yet some people have objected against this as a wrong way. But herein they are much mistaken; for if any force be increased, the effect will be increased in the same ratio; and there-

fore increasing the centrifugal force here, will increase the difference of the heights, in the very same proportion; as these quantities are very small. Upon this account, some people have unnecessarily run into long calculations, and only came at last to the same conclusion.

And the same argument holds good against such as have objected against finding the height of the tide (cor. Prop. 36. B. III.) by the rule of proportion, comparing it with the centrifugal force. For all forces of whatever kind will produce effects proportional to the *quantities* of these forces; and therefore he has rightly found the height of the tide, by that proportion.

But Sir I. Newton's explanation of the tides (Prop. 24. B. III.) does not please *Euler*, tho' he accounts for every circumstance thereof. He thinks ascribing these effects to the actions of the sun and moon, is recurring to *occult causes*, and therefore he had rather recur to *vortexes* for the explanation thereof; the notion of which has been confuted over and over. He denies the gravitation of bodies towards one another, because he cannot discover the cause of gravity; and therefore he will not allow it to have any thing to do with the matter, as being an occult quality. But he recurs to a principle that is more than occult, his incomprehensible vortices; which he thinks the tides are raised by; tho' he has not attempted to explain in what manner his vortices can do it.

He says, Sir I. Newton's account of the tides is not sufficiently explained, that any certain judgment can be formed whether it is true or false; that he did not explain the phœnomena of the tides but only darkened them. But certainly this man does not *know* what he writes, or does not *care* what he writes. Sir Isaac explained all the principal phœnomena thereof, and has shewn, that his

Theory

Theory is agreeable to obſervations both in nature and quantity. And all matters of leſs note may be eaſily deduced from his general Theory, by any intelligent perſon. But Sir Iſaac did not chooſe to erect any *pontes aſinorum*.

This author values himſelf very much for rejecting the principle of *attraction*, eſtabliſhed by *ſome Engliſhmen*, becauſe it is an occult quality. This profound Philoſopher will either know every thing or nothing; he will make no uſe of the forces of gravity, becauſe he knows not the cauſe of gravity. For the very ſame reaſon he would want to know the cauſe of that cauſe; and ſo he muſt know every thing in the whole chain of cauſes and effects, or he cannot be ſatisfied. This odd temper in ſome men ariſes from the pride of the human mind, which attempts to ſoar above its ſphere. They diſdain to know the little matter that is within their power to know; whilſt they are continually aſpiring at things that are without the reach of their knowledge, things for which they have no *faculties* ſuited to underſtand, and no *data* to determine. This is not to philoſophiſe, but to trifle.

As this Gentleman takes the liberty to ſneer at the *Engliſh*, he may pleaſe to ſtay and take this obſervation along with him, that if it had not been for an *Engliſhman*, he (and ſuch like) had known nothing. To him they are beholden for all theſe great and wonderful diſcoveries, which all the world acknowledge and praiſe him for, except theſe ſons of detraction, envy, and ingratitude. Diſcoveries that may be looked upon as the works of a Genius rather divine than human.

The *Engliſh* have found by experience, that by the power of ſome cauſe or other, all bodies are drawn or impelled towards one another, according to certain laws. And theſe laws are found out by obſervations, and a juſt reaſoning from the laws of

 motion.

motion. This cause, be what it will, they call *gravity*, without pretending to determine of what kind it is. If this author knew the physical cause of gravity never so well, he would be no wiser; for the effects of it would be just the same; and we can measure the effects of gravity, and from thence find the quantities, and proportions of the generating forces, without knowing what these forces consist in. So that the knowledge of such a cause would be only a useless speculation. But this author is so fearful of occult causes, that he dare not make use of their manifest effects; and condemns the English at once for believing their own eyes, that there are such effects flowing from this unknown cause, which they have agreed to call *gravity*. For it appears to them, that the effects thereof are all that they have any thing to do with. Sir I. Newton tells him more than once, *that he does not take upon him to define the kind or manner of action, or the causes or physical reason thereof*; but this author cannot take it in.

Some persons, when they can find nothing else to say, have cavilled about the term *gravity*, and so their objections are dwindled to a dispute about words, and not things. I suppose every thing in nature, that we have occasion to make the least use of in our discourse and reasoning, be it known or unknown, ought to have a name given it, to distinguish it from other things, and to convey our meaning to others. And thus the *cause* of the acceleration of bodies towards one another is expressed by the word *gravity*, and the action itself is called *gravitation*. The word *attraction* is used in the very same sense; that is, not in a *physical* but *mathematical* sense, which regards only the *quantity* of the cause. That these terms are the most proper that could be chosen, will appear from hence; the word *attraction* is taken from its most manifest quality; for a body

dy moving towards another with an accelerated motion, has all the appearance poſſible of being drawn towards that other by ſome inherent virtue in the other. The word *gravity* has evermore expreſſed the tendency of a heavy body to the earth; and therefore by parity of reaſon, will as properly expreſs the tendency of the moon towards the earth; or of the earth towards the ſun, or of any body towards another body.

Suppoſe this philoſopher, or any other of that ſort, was aſked to calculate the times, ſpaces, or velocities of falling bodies; which is the moſt ſimple caſe that can be propoſed about gravity. Would not any body juſtly laugh at him, if he ſtood to demur about it, and refuſe to calculate, till he knew whether their falling was cauſed by attraction, impulſion, or the rotation of a vortex? And would not he equally deſerve to be laughed at, that ſhould heſitate to calculate the motions of the moon, or of the earth and planets; or of the tides; upon the ſame account; when they are all acted upon by the ſame unknown cauſe of gravity?

But to return to the tides. This Gentleman (*Euler*) tells us, that Newton's method is erroneous, by which he found the ſea to riſe to the height of near two feet, by the ſun's force only. And ſays, that Newton found out this enormous effect, by comparing the ſun's force with the centrifugal force of the earth. This has been anſwered before; and certainly this Gentleman knows little about the nature of forces, if he does not allow that two equal forces, of however different kinds, will always have equal effects; and proportional forces, proportional effects, eſpecially in their naſcent ſtate. For it is not the *kind*, but the *quantity* of force that is to be regarded. Therefore Newton rightly found the ſolar tide near two feet, and the lunar tide $8\frac{1}{2}$ feet, agreeable to experience. But to ſhew you what ſort of a theory this Gentleman works by, he finds

finds the ſolar tide only half a foot, and the lunar tide $2\frac{1}{4}$ feet, in all not three feet; which all obſervations confute, and with it, his erroneous method of computation. I have met with no body yet, but what makes it at leaſt three times as much.

He alſo tells us, that Newton found out the forces of the ſun and moon by help of the tides, but he has not done it accurately. And yet Newton took in every circumſtance that could any way affect it; as may be ſeen in Prop. 37. B. III.

Having had occaſion to compare different kinds of forces with one another; I will venture to lay down this as a general Rule, that all forces whatever, whether attractive or impulſive, centripetal or centrifugal, or of what kind ſoever, if they be equal, they will produce equal effects. And therefore how idle muſt it be for theſe men to wrangle about the *kind*, when the *quantity* only is concerned in the effect, and can only be of any real uſe to us in our calculations. The enquiry after the *kind* and *modus* of action, is a phyſical, or rather a metaphyſical ſpeculation, the knowledge of which they can never come at.

It has likewiſe been objected by ſome perſons, that the two examples of Newton, for finding the tides, are ill choſen. But however, he had no more to chooſe on, and by their near agreement, it ſhews they were well choſen. *Euler* tells you, that at *Havre de Grace*, the greateſt and leaſt tides are as 17 to 11; and therefore the ſun's force to the moon's, will be as $17 - 11$ to $17 + 11$, or as 6 to 28; or as he makes it, as 7.13 to 28, which is about as 1 to 4, a proportion not very different from Newton's. *Dan. Bernoulli* ſays, that at St. *Malo*'s, the greateſt height to the leaſt, is as 50 to 15, which makes the ſun's force to the moon's, as 35 to 65, or as 7 to 13, not ſo much as 1 to 2; a concluſion utterly inconſiſtent with all other obſervations;

servations; which argues, that the observation has not been made with sufficient accuracy. However this is certain, that if any place can be improper for such an experiment, this place is; by reason of the very extraordinary tides. For here the tide being hurried up a long channel, growing continually straiter, it is forced up to an unusual height.

However, I cannot think that it signifies a great deal, where, or in what places, these experiments are made, provided the sea be deep, and have free access and recess to and from the place of observation. For tho' the tides be higher in one place than another, the sun and moon conspire alike to that; for if the water be accelerated in any degree by the moon's force, it will likewise be accelerated proportionably by the sun's force; so that the result will be nearly the same.

I cannot but be surprised, that in so material a point as this, no body has been sent purposely, to proper places to make observations of the tides. Since by this method only, the forces of the sun and moon can be determined to any tolerable degree of exactness. No celestial observations can assist us in this matter. Astronomy affords us no help. And there seems to be no way for us, to gain this great point, but this method by the tides. The forces of the sun and moon are so very small, in respect to the force of gravity, that no common hydrostatical experiment can shew us the least effects thereof. It is only in the tides that their effects become sensible.

The forces of the sun and moon reach to the very center of the earth, and act upon every point of the radius or column of water under them; and diminish the gravity of every particle thereof; and all these forces conspire together to raise the tide; and therefore the radius of the earth becomes the proper scale for all these forces to act on. And

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we cannot ſee the total effect of all theſe forces, unleſs we have a depth equal to the whole radius of the earth, or an extent of ſea of 90 degrees; and this effect or the tides, when at the greateſt, will then only amount to a few feet.

If I was to give directions for making obſervations of the tides for this purpoſe, I would adviſe to chooſe ſome place near the equinoctial, as on the coaſt of *Africa* or *America*. And the place would be moſt advantageous, where the ſun and moon are in the zenith at the firſt obſervation; and in the horizon, at the ſecond, for the ſpring tides. And one in the zenith and the other in the horizon, for two obſervations at the neap tides. And ſuch places ſhould be choſen where the ſea is of large extent, and deep; ſo as to communicate freely with the place, back and forward. And ſuch places will be beſt in a calm country, and performed in calm weather; and all circumſtances of weather ſhould be alike (as near as can be) at any two correſpondent times of obſervation, the ſyziges and quadratures. Theſe obſervations ſhould be made at the ſyziges and quadratures, both day and night. And alſo when the tides are higheſt and loweſt, which is three days after. But for ſome purpoſes, trial ſhould be made every day. And the buſineſs of trying at the ſyziges and quadratures, ſhould be continued for a twelvemonth. And more places than one ſhould be tried. Perhaps ſome iſlands in the middle of the ſea, as *St. Helena*, may be proper; for the weather is likely to be more uniform, than on the continent. Having thus gained a ſufficient number of obſervations, the beſt may be ſelected, and a mean ratio found, by which this matter will be finally determined.

There are ſome people that object againſt this method of finding the ſun and moon's forces, by the tides; and reckon it very precarious, and ſub-

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ject to many obstacles and intervening causes, by which the tides are perpetually influenced and disturbed; as if every thing had not its difficulties; the only disturbing cause is the wind. Yet they can tell us of no other method, but what is more precarious and more impracticable, and less exact. And so much for the tides.

The 39th prop. is about finding the precession of the equinoxes; against this, being a prob. of great difficulty, objections have been raised by several people, alledging that it is not truly demonstrated. Mr. *Simpson* is among this clan; and he absurdly makes the precession by the sun's force alone to be 21″ 6‴, which is double to Sir I. Newton's; the consequence of which is, that the motion by the moon's force will be only 29″. So that by this, the moon's force will be to the sun's, only as $1\frac{2}{5}$ to 1; yet he says in another place, that the moon's force to the sun's cannot be less than $2\frac{1}{2}$ to 1. So inconsistent and erroneous are his operations. But this has been taken notice of before in the Comment. But the moon's force must be greater than $2\frac{1}{2}$; for in all observations of the tides, that have been regularly made, the moon's force is 4 or more. And it is hardly possible in any observation of the tides, made with any tolerable degree of care, to miss near a half. They that would see the precession of the equinoxes truly calculated, will find it in Prob. 26, Sect. III. of my Fluxions.

Other objections of less moment I pass over, as their malignity and falshood will appear to every reader. Such as the absurd opinion that motion cannot be lost in the world; (concerning which, see my Mechanics, 4to. prop. 10. and cors.) Also the false opinions of these that deny that the composition and resolution of forces, are analogous to the composition and resolution of motions, which are their

their adequate effects. Their endless cavils against his method of demonstrating any proposition, which is the synthetical; preferring their own analitical methods as being shorter, which is no wonder. Their introducing into physical calculations, an obscure, precarious law or force, called *vis viva*; founded upon no certain principles. Concerning which see my Mechan. 4to. schol. to prop. 11, where the nature of it is unravelled. Their promiscuously using the words force and motion, for one another, which are as different as cause and effect; which induces no small obscurity into their writings. Their ascribing the invention of *Fluxions* to *Leibnitz*, contrary to the clearest testimonies; and some of them tell us, that, *not he that first found, but he that first published it, deserves the praise*. As if the publisher could have published it, at all, if the inventor had not first found it out. Here the inventor is robbed of his due praise, to give it to the thief that stole it. And in general, their aggravating every trifling slip, as a capital crime; and instead of praising him for what he has done (which is more than all the world ever did before) they dispraise him for what he *has not* done, or had not time to do; and lampoon him because his time or his knowledge was *not infinite*.

But if any body should ask, what any of these *Bravo's* have done since? The answer is, nothing at all, or less than nothing; they have been turning science backwards. For they have been doing nothing but undermining his principles, (tho' built upon the surest foundations,) to introduce their own chimerical hypotheses, that have nothing to support them but impudence, ignorance and presumption.

At the end of the Principia, Sir I. Newton has given us his thoughts of the deity. Here he shews that God is an *eternal*, *infinite*, and *powerful* being.

being. That all the frame of nature is owing to him, which he made, and governs. And from the ſimilitude of all the parts of the world, he ſhews that God is *one*. That he is a being acting with *council* and *deſign*, and with the greateſt *wiſdom*. That we have ideas only of his *attributes*, but know nothing of his *ſubſtance*; nor after what manner he acts or does any thing. Indeed we know nothing of the ſubſtance of any thing, much leſs of God. And as we cannot conceive of ſpace and time, but as neceſſarily exiſting; much more muſt we allow that God exiſts neceſſarily; and conſequently that he exiſts *always* and *every where*; and that not *partially* but *totally*. In ſhort, he has laid down the beſt metaphyſical notions of God, that can be met with any where.

But M. *Leibnitz* will not allow that God has in himſelf (much leſs other animals), any principle or power of acting, but as he is firſt acted on by ſome *motive*, which he thinks is to determine his actions. And in conſequence of that, if two equal ways of acting were laid before God, he could chooſe neither; but be like a balance, acted on by two equal weights, that would turn no way. But at this rate God is not a *free agent*, but a mere *patient*. He reckons it an *imperfection* in God to be able to chooſe *one* out of two perfectly equal things; becauſe he ſays there wants a *ſufficient reaſon*; and therefore he can chooſe neither. But it is certainly a *greater imperfection* to chooſe *neither*, than to chooſe *one*. It is a principle with him that God muſt have a ſufficient reaſon; which is true, but it is God's mere will that is the ſufficient reaſon. And how comes he to know what is a ſufficient reaſon to a perfectly free agent, that has it in his power to do any thing; as if man, who is no more than a *worm* in the creation, can preſume to know what is fitteſt to be done.

Sir

Sir I. Newton has aſſerted, that all the great bodies in the world move in free ſpace, which is unbounded and infinite. That all bodies have pores or empty ſpaces within them. But this author will not allow any *vacuum*, but will have the world be a perfect *plenum*. And he tells us alſo, that the world will continue for ever without any alteration (or mending); but he has taken particular care to prevent that, by introducing his *plenum*. The motions of the heavenly bodies muſt needs be retarded and ſoon ſtopt, in moving thro' denſe matter, tho' never ſo fluid. So that one of his ſuppoſitions is inconſiſtent with another; and no body could more effectually deſtroy his own hypotheſis, than he has done himſelf. He had had infinitely better chance for this ſuppoſition, if he had made the planets, &c. to move in a vacuum.

If God had deſigned every thing to ſtagnate, and be fixt in the world; then a plenum ſeems to be the beſt conſtitution for that end. But as all the operations of nature are to be performed by motion, which indeed is the beauty of nature; then placing them in vacuo, muſt needs be the beſt conſtitution, as the motions will then be free and unreſiſted, and the moſt durable poſſible. And therefore it is as neceſſary to have a *vacuum*, in which theſe motions are to be performed, as it is neceſſary to have *body* to perform theſe motions.

The ſame objections he has made againſt empty ſpace, he makes againſt time; for with him ſpace is no more than the order of co-exiſting beings; and time the order of ſucceſſive ones; but at this rate the words *nearer* or *further*, *ſooner* or *later*, ſignify nothing; yet both ſpace and time are meaſured by quantity, and therefore are themſelves quantities and conſequently real beings, which confutes his notions. And indeed common ſenſe is enough to determine this; for their exiſtence is ſelf-evident, even

even to the most ignorant; and cannot be made more evident by all the arguments in the world. Arguments in such cases, serve for nothing but perplexing things that are plain of themselves, and are often brought for that purpose. *Actions* and a *vacuum* are the first principles of the most ancient Philosophy; which is the same thing as to say, that God has made *bodies*, and *room* for them to act and to move in. For the whole world appears to our senses, to be nothing but *matter* and *motion*.

This learned Philosopher's argument against empty space is this; *every perfection which God could give to things, without derogating from their other perfections, he has given them. Suppose then an empty space, God could have placed matter in it, (which is more excellent than empty space); therefore he has done it, and consequently there is no vacuum.* Here he supposes that there is no excellency or perfection at all in motion; and so this world-maker will not agree to leave any room or free space for bodies to move in. And in this he is as dogmatical, as if he had been originally one of God Almighty's privy council. But had he gone one step further, he would have seen the absurdity of it. For God did not make the world to stand still; but to move after various laws and rules. And consequently when this acute Philosopher fills the planetary regions with matter, he *derogates from the perfections* of the other bodies, by destroying their motion, with his new matter. And in this case, and for this very purpose, empty space is more excellent than matter.

But the *ignis fatuus* that leads him astray, is the argument of *a sufficient reason*; when at the same time he is no judge at all what is sufficient. Yet this principle he brandishes about with no small ostentation and assurance. But it is of no more use in his hands, than the sword of *Achilles*, in the hands of an infant.

CONCERNING THE

OPTICS.

NO ſooner did Sir *I. Newton* publiſh his theory of Light and Colours, than he was attacked on all ſides. A doctrine ſo ſtrange, and contrary to all the received opinions about the nature of light, could not be admitted. This brought him into many frivolous diſputes, with injudicious people that had imbibed other principles. Yet he ſhewed the truth of his theory, by undeniable experiments, if his oppoſers had but had underſtanding enough to conſider them. He had read lectures before, in the Univerſity of *Cambridge*, concerning light and colours, which he was thinking of publiſhing; but the diſputes he was involved in, from what he had publiſhed in the Tranſactions, made him lay aſide the thoughts of publiſhing any more. And ſo theſe lectures were laid by in the Univerſity, and were not printed till after his death. And in this I think he was much in the right; for who would ever ſpend any time or labour, to make men wiſe againſt their wills; and eſpecially a ſet of men, that loved cavilling and diſputing better than truth and knowledge.

By reaſon of theſe ſenſeleſs diſputes, he would not agree to print his book of Optics for many years; and at laſt it was only printed in Engliſh, with a deſign it ſhould not go abroad, to create any freſh diſputes; but was afterwards tranſlated into Latin.

This

This book, tho' full of the moſt curious and profound diſcoveries in *Optics*, could not paſs without the moſt ſevere ſcrutiny; and I believe, hardly a propoſition in the book but what was diſputed by ſome raſh Critic or other. But as the proofs were moſtly experimental; as ſoon as people had a mind to leave their prejudices, and try theſe experiments fairly and impartially; they found them all true and agreeable to his theory, which at length began to gain ground; and all diſputes are now ſubſided.

As there are ſome things laid down in this book, without their demonſtrations; ſuch of them as are difficult, are calculated in my book of Optics.

Towards the end of this book Sir Iſaac tells us, the true method of philoſophiſing, his thoughts of the original and conſtitution of the world, and of the Deity. He tells us the world could not ariſe out of a *Chaos*, by the mere laws of nature, but muſt be the work of a powerful, everliving agent, acting with wiſdom and deſign. He tells us, that it appears from the phœnomena, that *there is a being incorporeal, living, intelligent, omnipreſent; who in infinite ſpace, as it were in his ſenſory, ſees the things themſelves intimately, by their immediate preſence to* [illegible]*——whilſt the images only of theſe things are carried thro' our organs of ſenſe into our little ſenſoriums, and are there ſeen by that which in us perceives and thinks.*

But Mr. *Leibnitz* catching at the word *ſenſorium*, and perverting Sir Iſaac's meaning, tells you, that Sir Iſaac makes ſpace to be the ſenſorium of God, as an organ, by which he perceives things, and thereby makes God the ſoul of the world, contrary to Sir Iſaac's plain declaration; whereas he ſpeaks only by way of ſimile (*as it were, in his ſenſory*). For ſuch is the imperfection of our notions concerning God, that when we would deſcribe any

of his properties or attributes, we can only do it by similitude, comparing it with something similar to it in ourselves; which surely cannot be taken in a strict sense. Nothing is more common than to talk of ideas in the mind of God, when we speak of his knowledge or omniscience. Yet it is certain God has no need of ideas or images (as we have), when he is present to the things themselves. And I think it was hardly possible for Sir Isaac to find out a fitter simile, than to compare *infinite space* with the *place* where *God* perceives all things; just as the brain is the place where *men* perceive the images (only) of things. For suppose the brain to be the *organ* of perception, it is nevertheless the *place where* these images are perceived. This misconstruing the words of Sir I. Newton, shews the captiousness of the man; especially when he is told that God is *a uniform being, void of organs, members or parts, that all things in the world are his Creatures, subordinate to him, and subservient to his will. That he is no more the soul of them, than the soul of a man is the soul of the species of things, in the place of his sensation.* All this, one would think, should stop his mouth.

This Gentleman calls God a *supramundane intelligence*, which is only an unintelligible term to quack with. For certainly he exists *in* the world, but after what manner is a secret to us. If God have any existence, he exists *in* space; to say that he exists *out of* space, is to say that he does not exist at all, because space is every where. So does he exist *in* time; for to exist *out of* time, or in no time, is not to exist. He exists then in all space and in all time, that is *always* and *every where*.

Sir Isaac tells us blind fate could never make all the planets move one and the same way, in concentric orbs; some inconsiderable irregularities excepted, which may have risen from the mutual actions of comets and planets

nets upon one another, and which will be apt to increaſe, till this ſyſtem wants a reformation.

Now theſe actions of the planets and comets on each other, and the conſequent irregularities produced thereby, are matters of fact, and evident from obſervations. And it is as evident that they will increaſe, from the ſame cauſes. Yet Mr. *Leibnitz* is ſo ignorant of theſe effects, that he takes upon him not only to cenſure Sir I. Newton for making theſe obſervations, but God Almighty for not making his work otherwiſe; and compares him to an *imperfect workman*, that has his *watch* to wind up now and then. He thinks it an imperfection in God, if he does not make the world ſo as to laſt in the ſame ſtate for ever. At this rate he may as well think it an imperfection in his Maker, if man does not laſt for ever, and live for eternity.

It is a bold aſſertion to ſay that God cannot make any thing to what degree of perfection he pleaſes, or to laſt as long as he pleaſes, or to reform, regulate or alter it at his pleaſure. Nothing will pleaſe this Gentleman but to make God to act *ad extremum*, as all neceſſary agents do, if they can be called agents. At this rate, not only the world muſt be made ſo perfect, as to laſt for eternity, and want no regulating, which is the thing he affirms; but even the bodies of men and other animals, for the ſame reaſon as was obſerved before, muſt laſt for ever, and never come to decay, or need any repairs; which is arguing againſt the cleareſt evidence of his ſenſes; and is a demonſtration of the abſurdity of his hypotheſis. So that in ſome caſes it is true (what ſome men have objected againſt ſcience in general), that the greateſt philoſophers are the greateſt aſſes.

Go, teach eternal wiſdom how to rule,
Then drop into thyſelf, and be a fool.

POPE.

His Doctrine of *Indiſcernibles* is altogether as wild a notion as the other of forcing God to do his beſt. His principle is this, that there are not in nature, two *real* abſolute beings, *indiſcernible* from each other, (that is, two ſuch things as are perfectly alike). For if there were, he ſays, God and nature would act without a reaſon; it is a thing contrary to the divine wiſdom, and therefore they do not exiſt. And he inſtances in two leaves or two drops of water. And he ſeems to build his opinion upon obſervation, which is, that if you ſeek never ſo long, you cannot find two leaves alike; and upon this he raſhly concludes, that it is impoſſible; becauſe God would not be wiſe in doing of it. However, if he can give no wiſer a reaſon than this, I doubt his principle will fall to the ground. For in the nature of things, what antipathy can God have againſt two equal or like things, any more than againſt two unequal or unlike things. It is impious thus to tie the hands of God, and vain in him to pretend to know what God's wiſdom will chooſe or will not chooſe. His attempt to know this is as ridiculous as it would be, in attempting to move the whole world with the ſtrength of a finger.

I believe the reader will hardly be ſatisfied, whilſt I am endeavouring to confute his notion if I do not advance a better ſolution. This I ſhall attempt as follows, by a very ſimple and plain caſe. Suppoſe the length of the leaſt ſort of leaves to be a, of the greateſt b; their difference $b-a$ or d. Now I hope it will be allowed me, that the quantity or length d is infinitely diviſible; and conſequently there is an infinite number of different lengths contained in the length d; and hence there may be an infinite number of leaves, and all of different lengths, in a gradual increaſe from a to b. This being ſettled, let us conſider the

number

number of leaves. The number of them on any one tree is but finite, and the number of all the trees in the world is but finite; for the sands of the sea are numerable. Therefore all the leaves in the world make but a finite number. Suppose then any one length be named for a leaf; it will be the odds of infinity to 1, that in a geometrical exactness, there is not a leaf of that length in the world; because the number of *leaves* being short of the number of possible *cases*; there will be an infinite number of cases, that do not happen; and therefore it is infinity to 1 against the happening of any one in particular. It follows likewise that, name the length of any one leaf, and it is the odds of infinity to 1, that another cannot be found *precisely* of the same length.

Now this calculation regards only the length, but there are other circumstances as variable, such as breadth, weight, colour, form, curvature, different arrangement of the veins, &c. and all diversified in quantities and ways infinitely various; therefore the odds will, by this means, be infinitely multiplied; so that it is next to impossible for such a thing to happen naturally. And here, we need not to have supposed infinite degrees of length; for if we had but assumed a very great number of lengths, each differing by the least conceivable quantity; yet by the infinite number of circumstances that attend it, the result would still shew that it is almost as infinity to 1, against such an event happening. And by the same reasoning one cannot find two stones alike, or two drops of water.

As to drops of water, there is not as great a variety as in leaves. Yet if you suppose but one heterogeneous particle; that single particle is capable of an infinite number of situations; and there may be 2, 3, &c. or any number of such particles;

from all which, the ſame concluſion will happen as before. And here we may obſerve, that the more compounded any body is, the greater is the improbability of this event happening.

Hence appears the true reaſon why ſuch events do not happen, and that is, the infinite number of *poſſible* caſes, and the finite number of *real* caſes; and upon the infinitely *ſmall* chance there is for its happening; but it is not at all from the impoſſibility of the thing; which may be made plain by this inſtance. Suppoſe there be 100 dice, and any one ſhould undertake to throw all the aces at once. The number of chances is ſo great againſt him, that in all probability, he would never do it while the world ſtands. Yet the thing is not impoſſible in itſelf; for the dice may be ſo directed by art, as to be thrown every time.

So now the whole myſtery is diſcloſed that ſeduced this learned Philoſopher to abridge the *power* of God, for fear of contracting his *wiſdom*. You may ſee that all the ſizes or forms of leaves are alike indifferent to God, and nature has the ſame chance for any length or form. But what is all this to do with the *power* of God? He can certainly make millions of things all equal and alike, if he has need of them in the creation, as eaſily as he can make them unequal or unlike. But the excuſe this Gentleman has for this chimerical opinion, is this, he ſays *two things* perfectly alike will not be two things, but only *one thing*. But this is an aſſertion equally ridiculous, for two things will be as diſtinct, if they differ but in one property, as if they differed in ten thouſand. Nothing can identify two things (*i. e.* reduce them to one), but exiſting in the very ſame place, at the very ſame time, and having all other properties the ſame.

Men are ſuch imperfect artiſts, they cannot pre-

tend

tend to make things equal or alike, to a geometrical exactneſs; but they can make them ſo near as they cannot diſcover any difference. But God can work perfectly, which men can only imitate.

God is a being ſelf-exiſtent, omniſcient, omnipotent, and omnipreſent. This being can do all things that are poſſible to be done, or that imply no contradiction. Theſe are his natural attributes, as has been mentioned before; and which Sir I. Newton has given a ſublime deſcription of, both here and in the Principia, which this *babbler* has thought fit to criticiſe, but with an ill grace. As to his moral attributes, wherewith ſome people have inveſted and complimented God Almighty; as his *Goodneſs*, *Mercy*, *Juſtice*, &c. theſe are not real properties, but are only conſequent from, and included in, his wiſdom. As for example, God cannot be ſaid to be infinitely good, as he may be ſaid to be infinitely powerful, or infinitely wiſe; but he is always ſo far good, as is conſiſtent with his wiſdom; and the like for his mercy, &c. As to our knowledge of God, we muſt content ourſelves with the knowledge of his natural attributes; for we know nothing further relating to him. It is a profound ſecret to us how he acts, how he perceives, or how he exiſts. Here he hides himſelf from us and will not be known. Nor ſhould we even know that ſuch a being exiſts at all, but from our own exiſtence, and the regular phœnomena that appear in the world, which bear witneſs to it. So that we are ſure there muſt be a firſt cauſe, from whom and by whom all theſe things have their original; as is ſhewn at large by our great author Sir *I. Newton.*

I believe the improvements that have been made to the *Science of Optics*, ſince he left it, are few or none; his queries are ſtill to ſeek, no body has inveſtigated any of them. One would think that

theſe

these cavillers againſt gravity, as an *occult quality*, would have taken care to have inveſtigated its cauſe, hinted at in theſe queries; but it remains as occult as ever, for them. They are more ready to diſpute than to act. To compleat theſe things, we ſhould have another *Newton*; but as that is not to be expected, we need not think that any improvements will go faſt forward.

As I have conſidered the principal objections that have been made againſt the *Principia* and the *Optics*, moſtly by foreigners. It cannot be amiſs if I take notice of ſome that have been made againſt his *Chronology*, by ſome in our own country; to ſhew how they *bite the file*.

An

An Account of ſome of the numerous Inconſiſtencies, contained in the Objections made by the Rev. Dr. Rutherforth, *Regius Profeſſor of Divinity in the Univerſity of* Cambridge, *againſt Sir* I. Newton'*s Account of the Argonautic Expedition.*

Eſt genus hominum qui eſſe primos ſe omnium rerum volunt, nec ſunt:

TERENT. *Eun.*

THE Profeſſor ſets off by telling us, that the *whole of his (Sir Iſaac's) argument, for ſettling the time of the Argonautic expedition, depends upon three principles.* 1. *That Chiron and Muſeus made a ſphere.* 2. *That they drew the Colures of the equinoxes thro' the 15th degrees of Aries and Libra, and that of the Solſtices thro' the 15th degrees of Cancer and Capricorn.* 3. *That they knew how to draw the Colures exactly.*

Here he begins by aſſerting a manifeſt falſhood; for the *whole* of his argument does not conſiſt in theſe three Articles. For Sir Iſaac ſtrengthens his argument by ſo many events, taken out of hiſtory, which all conſpire to prove the ſame thing, that no body (except they be ſtrangely bigotted, like this Gentleman) can refuſe his aſſent. But let us examine his three grand principles. 1. *That Chiron and Muſeus made a ſphere.* Now this is a point of hiſtory, and therefore ought to be taken as matter of

of fact, or else there is an end of all reasoning from historical relations. Yet our Doctor shews a strong inclination to suspect its truth, if it could be done; and seems to waver between two opinions, whether it was, or was not so. However, not being able to get over that, he tells us, that these figures drawn on the sphere, must relate to religion; *for delineating constellations for the use of sailors, could be of no service for the practice of justice.* Here one would expect that he should have shewn us how delineating constellations *not* for the use of sailors, could be of any service to the practice of justice; or what relation there is between *justice* and the *constellations.* But unluckily he has left this short. Now every body knows the use of the constellations in sailing: they had no other way at that time to guide their ships thro' the ocean, but observations of the stars. This use then is obvious; but the other use, which the Professor dreams on, is to seek. As to *Hippo* the daughter, who is allowed to have studied nature in the same manner as the father; the Dr. says, she shewed the will of the Gods, by her oracular answers, or *from the rising of the stars.* The use she made of her knowledge, was to teach the will of the Gods *from the motion of the stars.* This is enough surely to shew that they both studied *Astronomy.* But then this knowledge was applied, it seems, to the purposes of *Augury* or telling fortunes. But can any body (but this Rev. Doctor) be so dull, as to think that two of the greatest Astronomers of the age should apply ALL this knowledge, to such a trifling insignificant purpose; and not rather apply *some* of it least, to the nobler purpose of *Navigation,* for which it is naturally adapted, and very much wanted for that purpose. For without the knowledge of the stars, they could make no long voy-

ages;

ages; nor had they any other method at that time to guide a ſhip, but the help of the ſtars. Conſequently it was natural for the navigators to apply to the aſtronomers for that purpoſe. But the Profeſſor ſeems ſtill not to be content in allowing theſe figures to be the conſtellations; for, ſays he, *perhaps this may not mean to form conſtellations, but to divide the heaven into regions.* As if dividing the heaven into regions or ſegments, could teach the *riſing and motion of the ſtars. Muſeus,* he ſays, *was a poet, and writ a poem concerning the ſphere, but there was no ſort of reaſon to conclude that he was a practical aſtronomer.* I dare ſay, no body would ever draw ſuch a concluſion, but this ſagacious Doctor. For can any body in their ſenſes ſuppoſe, that any man will take pains to write a poem concerning a ſubject, that he knows nothing at all about. How muſt he deſcribe the circles of the ſphere, or ſhew the nature and uſe thereof, if he did not underſtand the ſphere? But all thinking men muſt rather believe that he knew what he was doing, that he underſtood the ſphere, and of conſequence muſt be an aſtronomer. I ſay, the conſequence is plainly this, that there was a ſphere in the time of *Muſeus,* or he could not write about it, and that he underſtood the ſphere he was deſcribing; and if he alſo made a ſphere, as hiſtory informs us he did, and was the firſt that made one; then this point is fully determined. But if it was only a poem, yet certainly if a man writes upon a ſubject (eſpecially a mathematical one) he muſt be allowed to underſtand that ſubject, whether he writes in verſe or proſe. Yet the Rev. Doctor will not allow *Muſeus* to be an aſtronomer, becauſe he was a poet; which is juſt as good a reaſon as this, Dr. Rutherforth is a *Parſon,* and therefore is no *Aſtronomer.*

He ſays further, *the accounts we meet with in the*

poets

poets (concerning the Argonautic Expedition) *is only its fabulous history. And the names of the constellations Sir Isaac has taken from this fabulous history.* The poets then cannot be trusted, even to give the names of things truly. But this writer is certainly at liberty to call any thing fabulous he pleases. He may reckon the whole fabulous, if he likes; and that the expedition itself is nothing but a fable. The poets give us a mixture of truth and falshood, or fables intermixed with truth. This Sir Isaac knew, and by his *philosophical sieve*, could tell how to sift out the true, and leave the fabulous part behind. Whilst such writers as this Professor are forced to take altogether by the lump; and not knowing how to separate them, call the whole fabulous. However, what Sir *I. Newton* has taken, is mostly from the Historians, who must be supposed to relate matters of fact, and not fables; and these he corroborates from the testimony of the poets. And as far as I can find, this Doctor is of opinion, that the Constellations were formed without having any names given to them; or else what should hinder the names from being transmitted down to posterity, as well as the Constellations themselves. Nay, they could not be described without their names.

The next objection this Rev. Doctor makes is, placing the constellation *Argos* in such a position, that the principal star *Canopus* could not be seen; and he thinks, *it should rather have been placed in the zodiac.* But any body may consider, that the constellations in the zodiac are of far greater use than any of the rest, as they serve to shew the motions of the sun, moon, and all the planets; and therefore these must be first of all placed; and this being so, he could not place *Argos* in the zodiac without displacing some of the rest, that had been fixt before; consequently it must be placed where

he

he could find a vacancy. In the poſition ſhe was placed, her maſts, ſails, and rigging might be ſeen. Beſides, it is evidently more natural to place the ſhip in the horizon, with part of her inviſible, as if under water, than have her ſuſpended in the air; as ſhe would have been, had ſhe been ſet in the zodiac, where ſun, moon, and planets move, But let us come to his ſecond principle.

2. *That Chiron drew the colure of the equinoxes thro' the 15th degree of Aries and Libra*, &c. Now this is a point of hiſtory, which is all the *data* allowed, and therefore muſt be granted. If this Profeſſor diſputes it, then he is diſputing with *Eudoxus*, and not with Sir *I. Newton*; for *Eudoxus* was deſcribing the ſphere of the antients. But this Doctor tells us, *that he was deſcribing the ſphere in his own time*; the conſequence of which is, that *Eudoxus* (and even *Thales*) muſt have lived 940 years before Chriſt, which is a manifeſt Falſhood. He tells us, that in deſcribing the colure of the ſolſtices, Sir Iſaac *leaves off, and omits a very material part, which Hypparchus adds.* This ſagacious Doctor, it ſeems, did not ſee the reaſon of this; Sir Iſaac had *data* enough without going any further, to determine what he wanted. And if the remaining deſcription was as it ſtood in *Eudoxus*'s time (as this writer ſays); then what had Sir Iſaac to do with that? for the poſition of the colures muſt be quite altered in near 600 years. But the Doctor ſeems not to underſtand, that many of the conſtellations had been miſplaced, and drawn otherwiſe in ſucceeding ages, and could not well be rectified; and ſuch as were rightly placed, or could be rectified, Sir Iſaac made uſe of. Beſides, if this Objector knew any thing in Aſtronomy, he would eaſily underſtand, that ſuch ſtars as are placed near the pole, or far from the ecliptic, are ſubject to great errors; for a ſmall one there, becomes a very

very great one at the ecliptic, and therefore were judiciouſly omitted by Sir Iſaac; and for this reaſon, he need make no wonder of theſe being omitted. Beſides, where the place of the colure is too generally defined, and a ſtar cannot be found to direct it with ſufficient exactneſs; ſuch *data* as theſe ought to be left out. For any body that knows what he is about, will chooſe clear *data* before ambiguous ones; while ſuch a judge as this Doctor muſt take all without diſtinction. In order to determine this point, Sir I. Newton, with uncommon ſagacity and judgment, has in either caſe, choſen *five* of the beſt defined, and moſt unexceptionable places, to found his calculations upon; which he has compleated with as much ſkill and nicety, as the ſolution of ſuch a problem deſerved. So that the method he has followed will be admired and approved by all proper judges; tho' it be condemned by ſuch ignorant, perverſe critics as this, that are too dull to ſee the reaſon and beauty of ſuch a proceſs.

By this judicious way of proceeding, he has found the mean places of theſe colures (from theſe five), to be *Taurus* $6^s\ 29^\circ\ 15'$, and *Leo* $6^s\ 28^\circ\ 46'$, which are ſo ſurpriſingly near, that the moſt ſcrupulous cannot expect any thing nearer; which is a full teſtimony of its truth. So that at a mean, the four cardinal points had gone back $1^s\ 6^\circ\ 29'$, which amounts to 2627 years. Nay further than that, thoſe very places which this Profeſſor ſays he left out, he ſhews that they all fall in the ſolſtitial colure, as *Eudoxus* had deſcribed them; and conſequently that this had all the characters of that colure, according to that deſcription. But this purblind critic either could not or would not ſee all this. This being all made plain, will he ſay that *Eudoxus* lived ſo many years ſince? Yet he ſays as much, when he ſays *Eudoxus* deſcribed the

ſphere

ſphere as it was in his own time. But as far as I can ſee, he will neither allow the colures to paſs thro' the middle of the ſigns in *Eudoxus*'s time, nor *Chiron*'s time, nor any time elſe; ſo ſtrangely full of contradictions is he.

What he ſays about the little Bear is nothing to the purpoſe. If it was not formed into a conſtellation in *Chiron*'s time, what had Sir I. Newton to do with it. And even if it was, it was too near the pole to make any calculations upon; but any way, *Eudoxus* could not deſcribe the path of the colure in other words. But if the Profeſſor would have done any thing to the purpoſe, he ſhould have calculated how far this ſtar was from the colure; and then a better judgment could be made. He likewiſe ſays, *it is impoſſible to conclude from this deſcription of the cardinal points, that Chiron placed them in the middle of the conſtellations*. Now had the Doctor made any calculations to ſhew this impoſſibility, or to ſhew how far any of theſe points deviate from the colure, he had done ſomething. But he ſeems to be better qualified for forging arguments, than making calculations. But Sir Iſaac has actually made calculations, and has found and demonſtrated all theſe deſcriptions to be right, by the ſurprizing agreement among all theſe obſervations; which is ſufficient to overthrow all the Doctor's wild gueſſes, and ſilence all his cavils at once. And indeed it is no wonder to find him miſtaken in a matter, which is quite out of his ſphere; when he miſtook in a matter within his own ſphere, about the nature of *Virtue*; till a Lady, Mrs. *Coburn*, ſet him right. *O Doctor!*

3. We come now to his third grand article, *whether they knew how to draw the colures exactly*. This is ſuch a childiſh objection, that I don't know what anſwer is fit to be given to it. If he means perfectly exact, there is no man at this day can do it.

For at this rate the colures are to be drawn precisely thro' ſuch and ſuch ſtars, which can be looked on no otherwiſe, than as ſo many mathematical points, which cannot be. All that can be concluded from hence is, that each colure, when rightly drawn, will paſs ſo near any ſtar, that the difference cannot well be diſcovered, by ſuch inſtruments as were then in uſe; or that ſuch and ſuch remarkable ſtars, come the neareſt thereto of any. But let us ſee how he argues upon this point. He tells us, *Eudoxus was deſcribing the ſphere as it ſtood in his time*; and in the very next words he confutes himſelf by telling us, *we are ſure that he placed them wrong, for we can prove that they were not in the middle of the conſtellations in his time.* Was ever poor Doctor ſo embarraſſed with an argument; he firſt tells us, that *Eudoxus* was deſcribing the ſphere in his own time; and immediately after, that ſuch a deſcription did not at all agree with the ſphere at that time. Would not any body but this Objector then conclude, that it was not deſcribed for that time, but for ſome other. Every circumſtance leads him, as it were, by the noſe, into a direct path, and yet he moſt ſtrangely miſſes his way. Inſtead of doing that, he has nothing to recur to, but that ridiculous ſhift, that *Eudoxus did not know how to draw the colures.* Here then it ſeems we have a famous Aſtronomer, and the greateſt of the age, that did not know how to draw one great circle perpendicular to another on the ſphere, he was ſo very ignorant. I know not what this Doctor may think, but I believe all the world will laugh at ſo idle a chimera. But the Doctor keeps fluttering about among theſe abſurdities, not knowing where to fix; but will readily agree to any impoſſible ſcheme, rather than to Sir *I. Newton.* Here one would think, if *Eudoxus* had miſſed his aim in drawing thro' one ſtar, he would ſcarce have

miſſed

missed two; and if he missed two, he would hardly miss three; but by strange ill luck, he has *missed* them all, according to him. And yet Sir Isaac has shewn that he has *hit* them all. So that this great Professor has made a fool of *Eudoxus*, or else of himself. But Professor *Simson*, of *Glasgow*, gives a quite different account of *Eudoxus*; he tells us, that it is the opinion of some learned men, *that Eudoxus was the author of the chief propositions in the 12th book of Euclid.* See his notes on B. 12. Euclid.

But to shew you how great an Astronomer this Professor is, he quotes *Hipparchus* for saying, that no secondary of the equator, could pass thro' all the places, before mentioned to be, in the colures. Now if this Gentleman knew any thing of the sphere, he would not at all wonder at this; because it is impossible, that the two great circles (the two colures) which were perpendicular to the equator in the time of *Chiron*; I say, it is impossible that the same two great circles should be perpendicular (or secondaries to) the equator, in the time of *Eudoxus*: For *Hipparchus* knew better. And this is another proof, that the sphere described was not for the time of *Eudoxus*, but for some other preceding time. And hence this Rev. Doctor need take no further pains to make *Eudoxus* an ignoramus, for it will all fall upon himself; all his arguments being founded upon his own ignorance; and all the consequences he draws from his absurd reasoning, must all fall at once. But where is his modesty, thus to fall foul of a person of such transcendent abilities as Sir Isaac; and who has obliged the world with so many eminent discoveries, and which were thought before to be beyond human art; and yet with no better success, as he himself appears to be no judge of it; and with no better design than to degrade that great man, who shines brighter for being cavilled at. For the same spirit

and genius that runs thro' his other noble works, are equally visible in this of his *Chronology*.

I think it needless to pursue this learned Doctor any further; it is plain he did not understand what he was writing about: every incident confutes him. But Sir Isaac stands firm, his plan is confirmed not only by the agreement among the observations, but also by a great many other proofs of a different nature, all conspiring to prove the same thing. And in such cases as this, which are not capable of mathematical demonstration; the concurrence of so many circumstances must make the thing so highly probable, as to take away all doubt and scruple, and give entire satisfaction.

After it was known that Sir *I. Newton* had writ a book of *Chronology*, a foreigner (one *Abbe Conti*) under the pretence of friendship to Sir Isaac, obtained a copy of the Chronicle or Index, with a solemn promise to keep it secret. But observe how he kept his promise; he communicated it to one *Souciet*, a jesuit, who got a bookseller at *Paris* to print it, without Sir Isaac's knowledge or consent; and with it a pretended confutation of the Chronicle, by the said *Souciet*. This was sent to Sir Isaac, who confuted every article of this pretended confutation. So that this may be said to be stolen from him, much after the manner that Leibnitz stole his Fluxions; but with a worse design, to involve Sir Isaac in disputes to oblige *Leibnitz*.

And the same *Souciet*, after Sir Isaac's death, writ another Tract against him. But Dr. Halley shewed him, that thro' his ignorance in Astronomy, he had made false calculations; and had assumed a wrong star, for the first star of Aries; and shewed him when all these things were rectified, the matter would be just as Sir *I. Newton* had described it. Thus you see how this great man has been used.

Of

Of Dr. BEDFORD'*s* CHRONOLOGY.

THIS Author is one of the ſame ſtamp as the foregoing, or rather more enthuſiaſtical; for if you will believe him, he can tell the very day when the moſt remarkable events in the beginning of the world happened. He promiſes a large book with maps, wherein, no doubt, he will inform you of the very day when the world began, where the garden of *Eden* was placed, where the Ark reſted, and how every Spot of Ground was peopled after the Flood, &c. And all ſuch difficulties as are without the power of Sir *I. Newton*, or any body elſe beſides. For in the Advertiſement to the preſent book, he tells us, that the ſun was created in the Equinox on a *Thurſday*, and the moon at full; that God ſpoke to *Noah* on a *Sunday*; that *Noah* ceaſed from bringing the living creatures into the Ark, on a *Sunday*; that God ſpoke to him to come out of the Ark on a *Sunday*; that God ſpake to *Abraham* on a *Sunday*; that *Solomon* dedicated the temple on a *Sunday*; that the angels ſung Glory to God, for the birth of Chriſt, on a *Sunday*, &c. &c. So that our Author, to be able to ſolve ſuch problems as theſe, without any *Data*, cannot be leſs than inſpired; and ſo I believe he is, with preſumption and vanity. But I think few people, who are thus glutted with ſuch impoſſibilities, will think it worth their while to penetrate any further into this profound Treatiſe; in which he is as confident, as if he had been preſent at every event. I ſhall therefore only give a few ſpecimens of his fine reaſoning.

He begins his book with a moſt fulſome *Encomium* upon Sir *I. Newton*, as a great Mathemati-

cian; tho' he was no manner of judge about it, as will be ſeen after. He tells you, that Sir Iſaac's Chronology was writ in his dotage; tho' Sir Iſaac expreſsly ſays, he did it when at *Cambridge*, and therefore when he was a young man; and he only reviſed it when he was old. And in reality it had been the work of his whole life-time, at his vacant hours. This Author tells us, the reaſon that Sir Iſaac would not publiſh it in his life-time was, that he was conſcious of his miſtakes and errors. And he may as well ſay, that his refuſing to print his *Optical lectures*, or his book of *Optics*, was becauſe he was conſcious of their being erroneous. So ready are ſuch authors as this to give falſe reaſons for things, and miſs the true; for the true reaſon was, to prevent waſting his time in uſeleſs diſputes, with weak and obſtinate perſons, not worth a diſpute, like this Author,

This Doctor ſeems to be under great concern, that Sir Iſaac's Chronology ſhould differ from the Scripture Chronology; but I believe it is rather for differing from three or four biſhops he mentions. For Sir Iſaac takes care all along to connect his account with that of the Scripture; tho' perhaps not according to the ſenſe of this Author. And becauſe Sir Iſaac differs from all other Chronologers, therefore he thinks it muſt needs be falſe, and ſo he condemns it at once, becauſe not agreeable with him and his four Biſhops.

It is no news to tell us, that Sir *I. Newton*'s Chronology differs from all the world; it was Sir Iſaac's profeſſed deſign. He attempted to correct the antient Chronology, which no body elſe could do, or at leaſt has done. But this Rev. Doctor gives us nothing but a *hotch potch* of the common Chronology, with all its inconſiſtencies and errors; which he knew not how to get rid on, or to mend. This he intermixes with his own wild conjectures;

and

and a romantic account of his own, taken at random from all sorts of authors. But let us see how he goes on.

Arguing against Sir *I. Newton* about ship building. He tells us, that *the first pattern for ships was the ark, which was made square, that it was not designed for sailing, but to lie on the water.* How comes it then to be a fit pattern for ships, that are to sail from one place to another? And he tells us, that *tall and long ships were probably invented by Ham, who having been in the ark, could not be ignorant of navigation.* As if a man that was close shut up in a trunk, which was to lye upon the sea, could from that situation, conceive any notion of navigation: so little knows he what *Navigation* means.

Sir *Isaac* says, the Chronology of things done in Europe above 80 or 100 years before *Cadmus* (who invented letters) cannot be admitted. This Doctor replies, *and the history of Moses, of the things done above* 100 *years before his time, will be destroyed by the same argument.* Here he puts *Moses* upon the same footing as other common historians, he owns no difference of circumstances.

Sir Isaac tells us, that *Sesac* left geographical tables of all his conquests at *Cholchos:* and there *Geography* had its rise. This Objector has nothing to answer but, *it is very seldom that such arts are found out by martial men, who have something else to do.* Here is another instance of his ignorance; for can any man invade a country, but he must know or find out the situation of places, how they are bounded by one another, or by the sea, &c. And the knowledge of this is *Geography.* And his ignorance is still further confirmed in this, where he makes *Sibor* not to be the *Nile*, but a river in the country of the Amalekites, where there is no such river. When he writ this, he certainly was *not capable of thinking.*

Sir Isaac says, the temple built by *Menes* could not be above two or three hundred years older than *Psammiticus* that finished it. This Doctor answers, *Chronology may be reduced still further; for* Herod *the great, who was alive at the birth of Christ, finished* Solomon's *temple; and therefore it could not be above two or three hundred years older than the reign of* Herod. So that this blundering writer cannot tell the difference between *finishing* and *repairing*. There are endless examples of such wretched stuff as this given for reasons and arguments, and hardly any thing better.

His representing Sir *I. Newton as not capable of knowing what was demonstration and what was not*; is very conformable to the sentiments and disposition of such a brute as this, that has not the least idea of the nature of a demonstration. Now for some strokes of his astronomical genius.

The first thing of this sort that he presents us with, is the calculation of the sun's place 2066 years before Christ, which place he falsely makes in *Capricorn* 14° 8′ 19″; which should be *Sagitary* 13° 23′ 20″. The Egyptian year did not take place 1319 years before *Nabonassar*; and if it did, this absurd calculation proves nothing at all. Yet from this, he makes the distance of the sun from the tropic only 45′, when he is 21 days from it, by his own calculation. Nor does it appear by what sort of legerdemain, he makes 45^m answer to 21 days. Also it may be observed, from his process, that he makes the Julian year take place from the Era of *Nabonassar*; which is another blunder.

When the Julian year begun, he tells us, the beginning of the year was, when the sun was in Capricorn 8° 50′ 57″, and its distance from the tropic 18′; so by this, the tropic was in about 8° 33′ of Capricorn, which is an absurdity.

He makes a remark from *Scaliger*, that the colure

lure could not paſs thro' the tail of the ſerpent, altho' there is no calculation made about the ſerpent; ſo that this is nothing to the purpoſe. Yet he concludes that the colure had been placed wrong, juſt the ſame as Dr. *Rutherforth* ſays, an aſtronomer as wiſe as himſelf.

This author tells us, *the mariners at ſea find out the latitude of the place, not from the exact height of the ſun at noon*; and from ſuch ſtuff as this, he tells you that *Chiron* (or ſome other) *would place the ſummer ſolſtice and its colure, not at the entrance of Cancer itſelf, but before it.*

In quoting *Hipparchus*, he tells us *Hipparchus well knew there was* 1100 *years between the time of the argonautic expedition, and the diſcovery of the motion of the equinox.* But certainly the author does not *well know* what he writes, for if *Hipparchus* had known this ſo well, he would have told us ſo. And all the conſequence he draws from this ſuppoſition is nothing but a heap of abſurd nonſenſe.

The next aſtronomical flouriſh he makes, is to calculate the time of *Arcturus'* riſing; but before he begins, he has his way to pave with ſo many *if's* and *and's*, and *ſuppoſitions*, that the reader muſt be ſurfeited before he begins his calculation. Then he aſſumes the lat. of *Cuma* (but it was *Aſcra* where *Heſiod* lived), to be 38° 54′, which ſhould be 39° 20′ at leaſt, but no body can tell exactly; but a ſmall error will do him a deal of ſervice, and he can apply it his own way. In calculating from his data, he finds the declination of *Arcturus* an. 870 before Chriſt, to be 34° 25′, which ſhould be 34° 19′ 32″; and his right aſcenſion 179° 32′, which ſhould be 178° 21′; and ſo finds *Arcturus* at ſun-ſet, 2° 56′ under the horizon, which ſhould be 1° 46′, by his own data, if he had wrought right. Now this eminent aſtronomer, or his computer,

puter, has made no manner of allowance for refraction; which being done, it will bring *Arcturus* very near the horizon, as it ought. Nor does he take notice of refraction in the rest of his sublime calculations that follow. But to make all clear, he tells us, that the number 60 may stand for 55, or 65, &c. which is a *poetica licentia*, that I did not understand before.

This doctor tells us Sir Isaac's whole book is built upon this, that *Chiron* made a sphere, &c. But this is a most impudent falsehood, for the whole book is not built upon this; it stands upon many more pillars, which this writer, with all his sophistry, is not able to demolish. Whether *Chiron*, or whoever made the sphere, it will be the same thing; if it was made at that expedition, or a little after, as there is no room to doubt.

This Rev. Doctor contends that the ages of men were longer in ancient times than at present; and therefore he thinks Sir Isaac has made them too short. But he cannot make it out, that they were any longer within the bounds of this chronicle. And *David* will be against him; for he tells us, not of one particular man's age, but in general, that the age of man is 70 or 80 years, as at present,

And the like objections he makes against the reigns of kings; tho' Sir Isaac has confirmed the thing by so many catalogues of vast length. And this Author's picking out short catalogues of greater lengths, whether of ages or generations, is nothing to the purpose. He may as well take one age or one generation, as a standard of all the rest. But this is not doing justice to the cause.

This worthy Doctor is so enraged, that Sir I. Newton's Chronology is opposite to all the world besides; that he damns it at once by wholesale; and falls into such a strain of raillery as is hardly to

be

be matched in all Billingsgate. He can talk of nought for several pages together but *contradicting the Scriptures*. This system of *Chronology* giving men a *dismal tincture*, rooting-out of *religion*, bringing in *infidelity*, that such *poison* ought not to go abroad; the *Devil's* spite against the *sacred oracles*, and the *mustering* of his *forces*, *blasphemies*, *heresies*, liberty for *new* religions, *Atheism*, *Deism*, *Hell torments*, man's *immoralities* and *debaucheries*, attacks on *Christianity* and Scripture *Chronology*; *smoaks* arising out of the *bottomless pit*, that *darken the sun and the air*, &c. &c. and, I believe, by the time that the reader has got thro' it all, he will be nearly choaked with all this smoke. And all this Rant is, because men will not swallow every stupid thing, that such men would cram down their throats. After so pathetic an harangue, no body need have any doubt about his pacific disposition; this pious Doctor, I believe, will hardly send any man to heaven, that should presume to give any credit to such a vile book as Sir *I. Newton*'s Chronology. But I understand not why all this stuff is introduced here into a book of Chronology; except this pious Doctor meant to call down *fire from heaven* upon all such wicked persons, as should take the liberty to differ from him in *Chronological matters*, as well as in *Religion*.

He frequently mentions Sir *I. Newton* as a great and eminent Mathematician, nay, the greatest in the world. Yet he dare not trust him with the solution of certain astronomical problems; but attempts to give more exact solutions to them himself. At this rate he must be a greater mathematician, and a *greater* than the *greatest* in the world. Or perhaps Astronomy must rather be reckoned a part of Divinity, than of Mathematicks.

However compleat he may be in the mathematical arts, he is certainly compleat in the art of

Squab-

Squibbling. For example, Sir Iſaac tells us, that the obſervations of the ancients were but *coarſe*. This word is of infinite ſervice to him, and he chews it over and over. By the *magic* of this word he can make any error, great or ſmall, ſerve his turn. For if he miſſes his mark never ſo far, he can always account for it from the *coarſe* obſervations of the antients. This with him is a word of great latitude, and has no bounds.

I ſuſpect what irritates his temper moſt of all is, that this regular ſyſtem of Sir Iſaac's is likely to overturn his great *Babel*; which makes him labour with might and main to ſecure it; but with more *pains* than *prudence*. For there is as much difference between them, as between a regular and beautiful garden, and a confuſed wilderneſs.

But to take my leave of *this Author*, this renowned Champion for vulgar errors; Sir Iſaac having proved *Oſiris*, *Bacchus*, and *Seſoſtris* and *Seſac*, to be all one and the ſame perſon. As each of them was king of all *Egypt* at the very *ſame* time, and by a great many other circumſtances common to them all. This acute Reaſoner argues thus; *there were three famous men in the world, Hannibal, Belliſarius, and Churchil; they were all very eminent generals, remarkable for their courage and conduct, led their armies into foreign countries, fought many battles, took many towns, &c. therefore they are the ſame with the Duke of Marlborough.*

But pray ſtop a little Rev. Doctor, did all theſe men go into the *ſame* foreign countries, fight the *ſame* battles, take the *ſame* towns, &c? if not, where is the ſimilitude? Now, Sir, as ſoon as you can make out, that theſe four men were each a general of the *whole* army, at one and the *ſame* time, in one and the *ſame* place (as the parallel requires); then in ſpite of your *Logic*, they will be one and the *ſame* man, or rather four different names

names for the *same* man. This Doctor is certainly very unhappy in drawing parallel cases, to chuse such where all the circumstances are different, instead of being the same or like. But for all that, the conclusion will be the very same with him, whether the circumstances be the same or different. Does not his scarlet blush at this! Could any body of common sense offer such childish things to the public. He certainly has a mean opinion of his readers, to think they can be thus imposed on. Yet all his parallel cases are of the same kind.

FINIS.

ERRATA.

Menſuration, p. 104, l. 2b. *read* $\frac{.7854}{3} \times$ CD, is the

Surveying, Preface, p. ii, l. 3b. 4b. *read* μίγμα.

ERRATA.

b ſignifies, *from the bottom.*

pag	line	read
5	2b	SA or TA;
6	0	pa. 6.
9	7b	$Sp^3 \times pv^3 =$
	6b	And $SP^3 \times PV^3 =$
10	1	round R, in P : :
	2b	rectangle *uPv*.]
	1b	$Qv^2 + uPv =$
11	2	$Pv \times uV + uPV = VPv$
	4b	*Conics Prop.* 24. *Ellipſis*)
		perpendicular SY
	3b	(ſee alſo Cor. 7.)
12	1	dele *fig.* 5.
	12	[Pr. 21.]three—lines. If
	21	Lem. 16.
	24	in the ſame
13	6	fig. 7.
	18	PI : : ſine of
	22	tangent at A,
14	6	dele fig. 11.
	10	fig. 11.
	11b	And *kn* : *ga* : :
	3b	O*e* : OD : :
15		dele *fig.* 13.

pag	line	read
16	1b	IH : HF : : (*i*X : XY
17	1	dele fig. 20.
19	3b	: : *s* : PM =
29	13b	*d* in the ſame
	2b	*de*, *fg* touch
	1b	from C on
30	3	of B in *d* : :
	14	thro' $\beta\gamma$
	16	A as $C\beta$
	24	: : $C\gamma - \gamma\delta$
87	1	AZ, C*z*
105	15	Iμ : : $p\sigma$:
109	16	$\frac{G - C}{G - \gamma} Q =$
Pl.	I	fig. 6. for u write n.
		fig. 4, 11, 12, for e write c.
		fig. 13. draw the prick lines OB, OeD.
Pl.	III	fig. 29. for n write w.
		fig. 36. for g write q.

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